SFT Inspired Cosmology

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Model

$$S = \int d^4x \sqrt{-g} \left\{ \frac{m_p^2}{2} R + \Phi F(\Box)\Phi - V(\Phi) \right\}$$

Superstring inspired

$$F(\Box) = e^{\tau\Box}(-\Box - \mu^2), \qquad m_p^2 = \frac{1}{8\pi G}$$

τ is a parameter from SFT

$$V(\Phi) = \frac{\varepsilon}{4}\Phi^4$$

FRW:

$$\frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$$

Main messages

- Eqs with infinite number of derivatives:

- On the half line in additional to the usual initial data a new arbitrary function (external source) occurs.

- Stretch of the kinetic energy

Outlook

i) Applications of nonlocal SFT eqs

to cosmological inflation.

ii) How to understand $F(\Box) = e^{\tau\Box}(-\Box - \mu^2),$

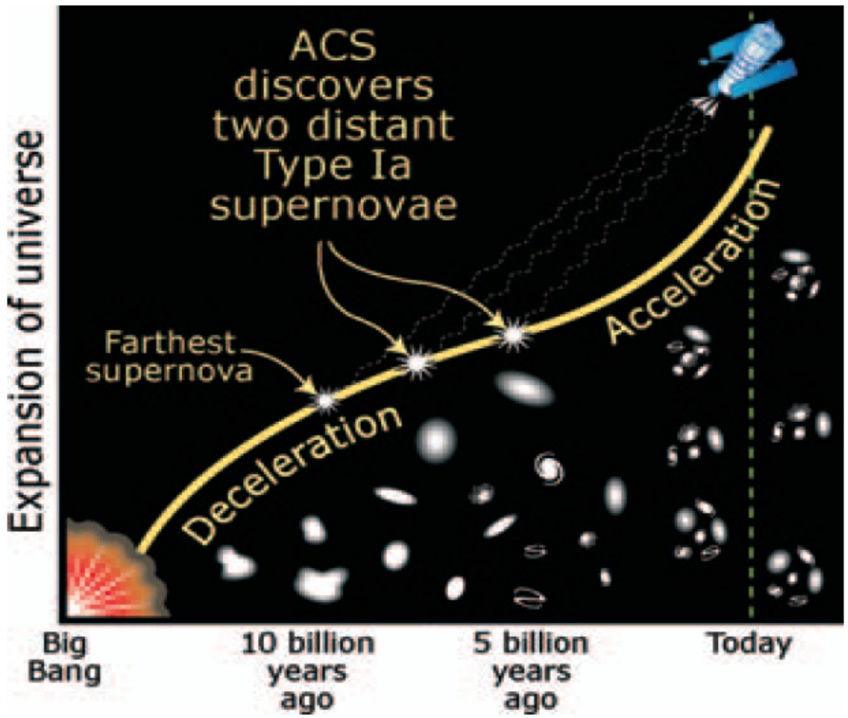
iii) Why half-axis ?

iv) Possible approximations

Brief History of Universe

- 14 Billions years:
- From Big Bang to Life on Earth

Brief History of the Universe



First Big Bang.

Then inflation.

Inflation Scenario

- The cosmological observations show that the Universe is almost flat and a density perturbations are scale invariant, Gaussian and adiabatic.

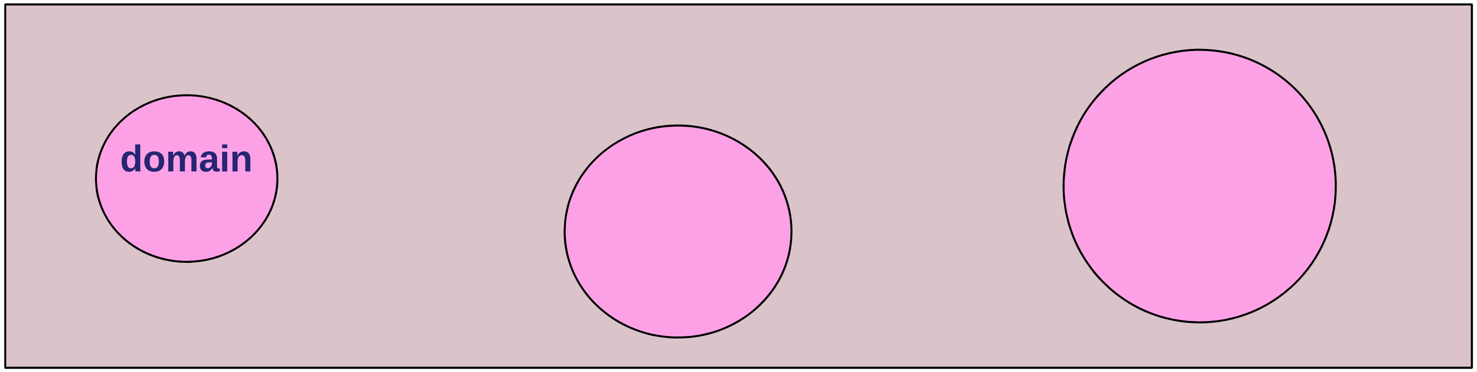
- The simplest explanation of these observations is provided by the slow-roll infation driven by a scalar field (Guth, Starobinsky, Linde) inflanton.

- The basic ideas of the chaotic infation scenario are very natural and general.

Inflation

- It is assumed that the early universe initially consisted of many domains with chaotically distributed scalar field.

Where we are?



Anthropic Principle

- According to the anthropic principle there are many universes and our universe is just one of them better suitable to support life as we know it.

FRW cosmology with one (local) scalar field

$$S = \int d^4x\sqrt{-g}\left\{\frac{m_p^2}{2}R + \Phi \,\Box\, \Phi - V(\Phi)\right\}$$

FRW:

$$\Box = -\partial_t^{\,2} - 3H\partial_t \qquad m_p^2 = \frac{1}{8\pi G}$$

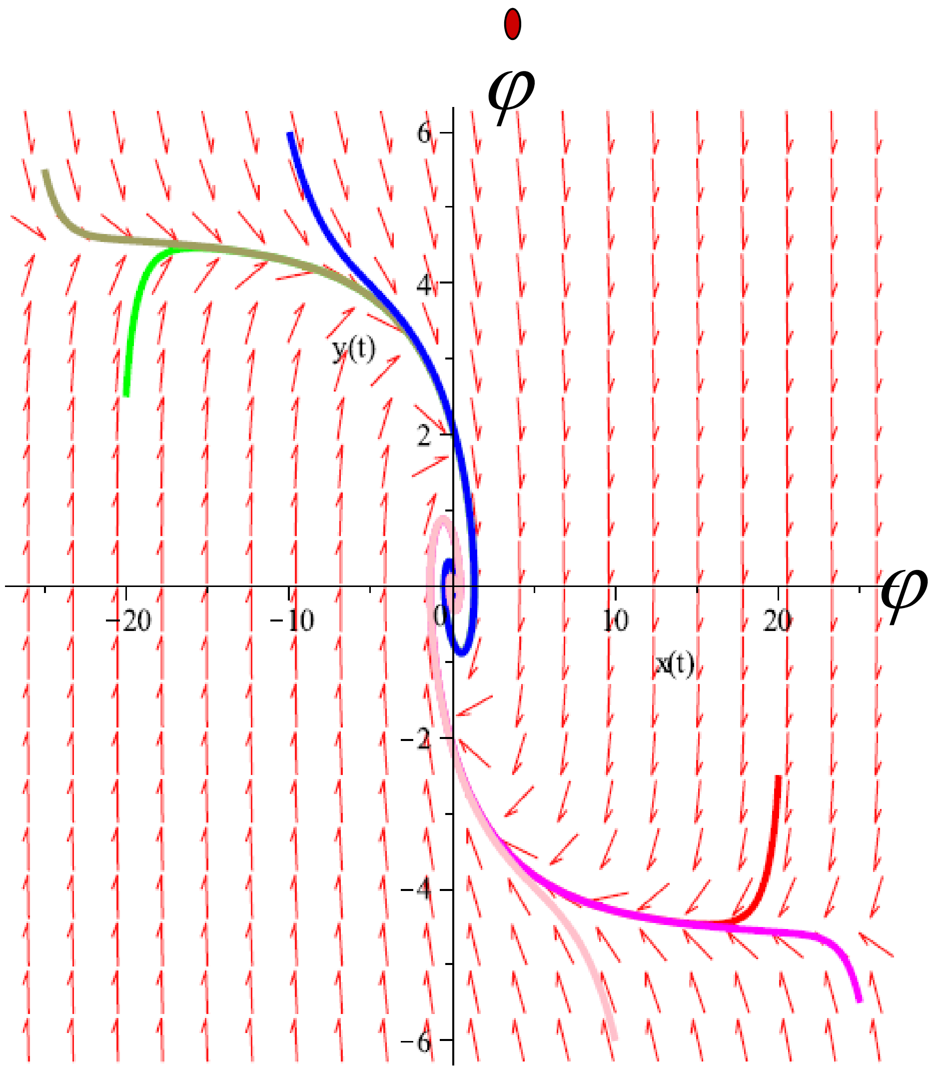
$$\ddot{\phi} + 3H\dot{\phi} = -V'_\phi$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

Simplest case $V(\phi) = m^2\phi^2/2$

$$\ddot{\phi} + 3\sqrt{\frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{m^2}{2}\phi^2\right)}\,\dot{\phi} = -m^2\phi$$

The inflaton flow in phase space (3 eras of the inflaton evolution)



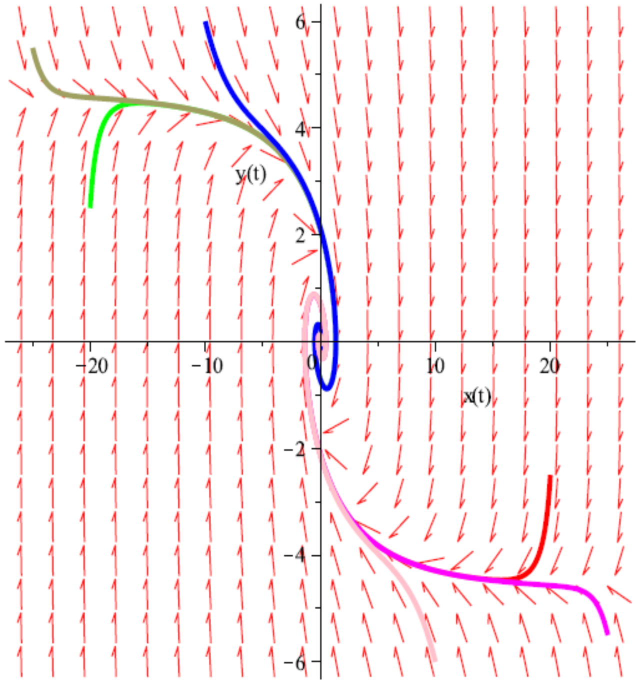
Chaotic inflation:

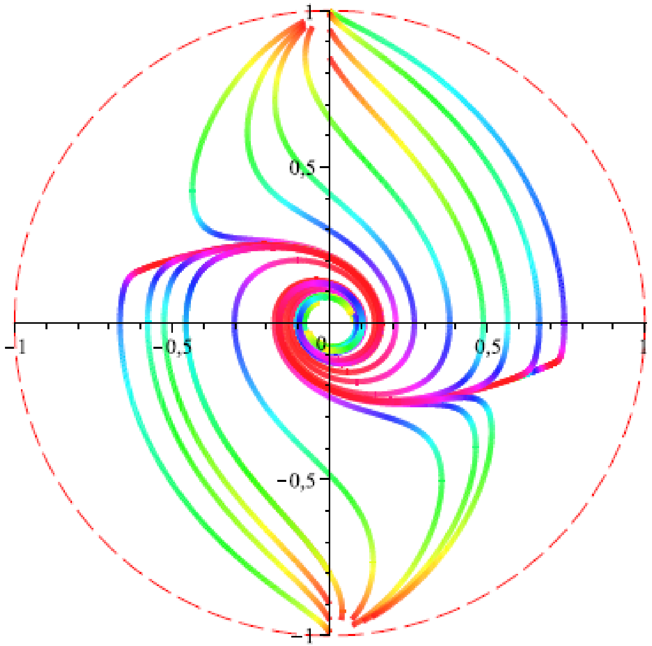
- The inflaton reaches very fast the attractor
- The inflaton moves along the attractor
- Oscillations near the zero

Problems with chaotic inflation:

- Large fields
- Non inflaton in the list of SM particles
- **Cosmological singularity**

Cosmological singularity





$$\phi = const - \frac{1}{\sqrt{12\pi G}} \ln t$$

$$H \simeq \frac{1}{3t}, \qquad a \sim t^{1/3}$$

V. A. Belinski'i, L. P. Grishchuk,
Ya. B. Zel'dovich, and I. M. Khalatnikov,
Zh. Eksp. Teor. Fiz. 89 (1985)346;
BGZK(1985)

Inflation as dynamical system with 3 DOFs in the unit ball

Dimensionless variables

$$X = \frac{\sqrt{12\pi}}{3m_p}\phi$$

$$Y = \frac{\sqrt{12\pi}}{3\mu m_p}\dot{\phi}$$

$$Z = \frac{H}{\mu}$$

$$\tau = \mu t$$

BGKZ-change of variables

$$X = \frac{\rho}{1-\rho}\sin\theta\cos\psi,$$

$$Y = \frac{\rho}{1-\rho}\sin\theta\sin\psi,$$

$$Z = \frac{\rho}{1-\rho}\cos\theta$$

Cosmological Singularity

- Classical versions of the Friedmann Big Bang cosmological models of the universe
  contain a singularity at the start of time.

Proposal: restrict ourself by considering the time variable t running over **the half-line** with regular boundary conditions at $t = 0$.

Cosmological Daemon

I. A., I.Volovich, JHEP 08 (2011)102, arXiv:1103.0273

- Nonlocal string field theory equations with infinite number of derivatives.
- We use the heat equation method and show that on the half-line in addition to the usual initial data a new arbitrary function (external source) occurs that is called the daemon function.
- The daemon function governs the evolution of the universe similar to Maxwell`s demon.
- In the simplest case the nonlocal scalar field reduces to the usual local scalar field coupled with an external source.

Nonlocal operator of SFT-type

$$F(\Box) = e^{\tau\,\Box}(-\Box + \mu^2), \quad \Box = \partial^2_{tt} - \partial^2_{x_i x_i}$$

On space of homogeneous functions

$$F(\Box) = F(\partial_t^2) = e^{\tau\,\partial_t^2}(-\partial_t^2 + \mu^2),$$

$e^{\tau\,\partial_t^2}$ **as solution of the diffusion equation**

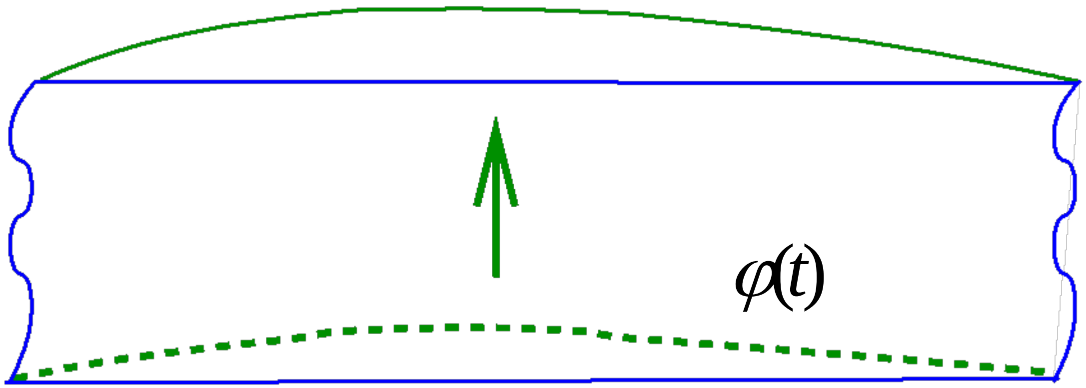
$$\Psi(\tau, t) = e^{\tau\partial_t^2}\varphi(t) \iff \begin{cases} (\partial_\tau - \partial_t^2)\Psi(t,\tau) = 0, \\ \Psi(t,\tau)|_{\tau=0} = \varphi(t). \end{cases}$$

The Heat Equation on the Whole Line

$$(\partial_\tau - \partial_t^2)\Psi(t,\tau) = 0,$$

$$\Psi(t,\tau)|_{\tau=0} = \varphi(t).$$

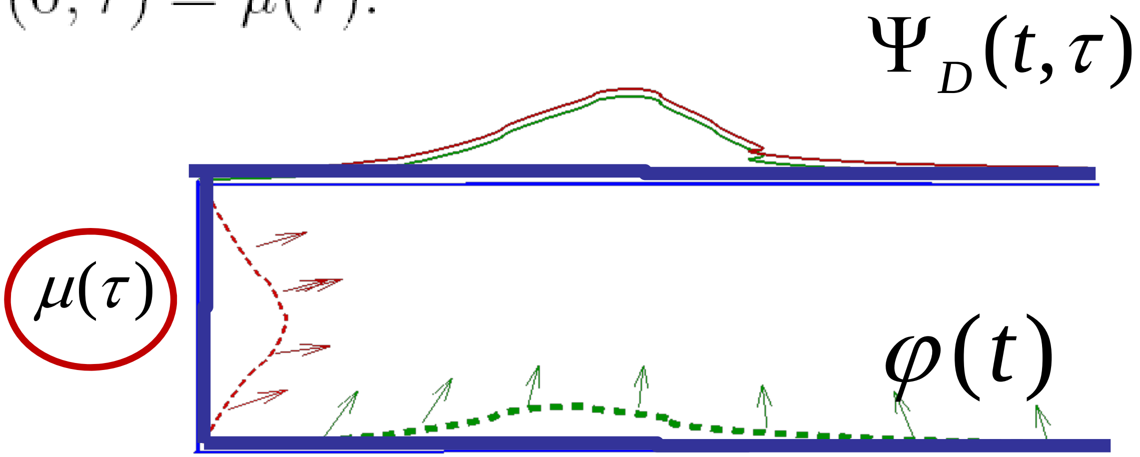
$$\Psi(t,\tau) \equiv e^{\tau\partial_t^2}\varphi(t) = \mathcal{K}[\varphi](t)$$



$$\mathcal{K}[\varphi](t) \equiv \frac{1}{\sqrt{4\pi\tau}} \int_{-\infty}^{\infty} \varphi(t')\, e^{-\frac{(t-t')^2}{4\tau}}\, dt'$$

The heat equation of the half-line

$$\begin{cases} \frac{\partial}{\partial \tau}\Psi_D(t,\tau) = \frac{\partial^2}{\partial t^2}\Psi_D(t,\tau), \quad t>0, \quad \tau>0, \\ \Psi_D(t,0) = \varphi(t), \\ \Psi_D(0,\tau) = \mu(\tau). \end{cases}$$



$$\Psi_D(t,\tau) = \frac{1}{\sqrt{4\pi\tau}}\int_0^\infty \varphi(t')\,[e^{-\frac{(t-t')^2}{4\tau}} - e^{-\frac{(t+t')^2}{4\tau}}]dt'$$

$$+\frac{t}{\sqrt{4\pi}}\int_0^\tau \frac{\mu(\tau')}{(\tau-\tau')^{3/2}} e^{-\frac{t^2}{4(\tau-\tau')}}\,d\tau' \equiv J(t)$$

Semantic remarks

J(x) – Daemon external source.

- **According to Plato: daemons are good or benevolent "supernatural beings between mortals and gods"**
- **Judeo-Christian usage of demon: a malignant spirit.**
- **Socrates‘ daimon is analogous to the guardian angel.**

Profit from the Daemon external source

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_P^2}{2} R + \frac{1}{2} \phi \Box \phi - U(\phi) + J\phi \right\},$$

J(x) – **Daemon external source**

$$\epsilon = \frac{M_P^2}{2} \left(\frac{U'(\phi) - J}{U(\phi) - J\phi} \right)^2$$

$$U(\phi) = m^2\phi^2/2 \,; \epsilon = \frac{M_P^2}{2\phi^2} \left(\frac{m^2\phi - J}{\frac{m^2}{2}\phi - J} \right)^2 = \frac{M_P^2}{2\phi^2} \frac{\delta^2}{(\frac{m^2}{2}\phi - \delta)^2}$$

$$\delta = m^2\phi - J$$

Nonlocal FRW Equations

$$(\Box + \mu^2)e^{-\tau\Box}\Phi - \varepsilon\Phi^3 = 0 \qquad \Box = -\partial_t^2 - 3H\partial_t$$

$$\underbrace{(\Box + \mu^2)e^{-\tau\Box}}_{F(\Box)}$$

$$3H^2 = \frac{1}{m_p^2}T_{00}$$

$$F(\Box_g) = \sum_{n=0}^{\infty} f_n \Box_g^n$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \left(E_{\mu\nu} + E_{\nu\mu} - g_{\mu\nu}\left(g^{\rho\sigma}E_{\rho\sigma} + W\right)\right)$$

$$E_{\mu\nu} \equiv \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \partial_\mu \Box_g^l \phi \partial_\nu \Box_g^{n-1-l}\phi \qquad W \equiv \frac{1}{2}\sum_{n=2}^{\infty} f_n \sum_{l=1}^{n-1} \Box_g^l \phi \Box_g^{n-l}\phi - \frac{f_0}{2}\phi^2 + V(\phi)$$

Solutions to Nonlinear Nonlocal Equation on the Whole/Half-line

- Dirichlet's daemon **without** source

$$e^{\tau\partial^2}(\partial^2-\mu^2)\phi(t)=-\epsilon\phi^3(t)$$

$$\phi(t)=a\sinh(\Omega\,t)-\frac{\epsilon a^3}{32\mu^2}e^{-9\lambda\Omega^2}\,\sinh(3\Omega\,t)+\dots$$

where $\Omega^2-\mu^2-\frac{3}{4}\epsilon a^2 e^{-\tau\Omega^2}=0$

$$E\;=\;\frac{a_1^2}{2}\Omega^2 e^{\lambda\Omega^2}+\varepsilon\frac{3a_1^4}{8}\left(\lambda\Omega^2-\frac{1}{4}\right)$$

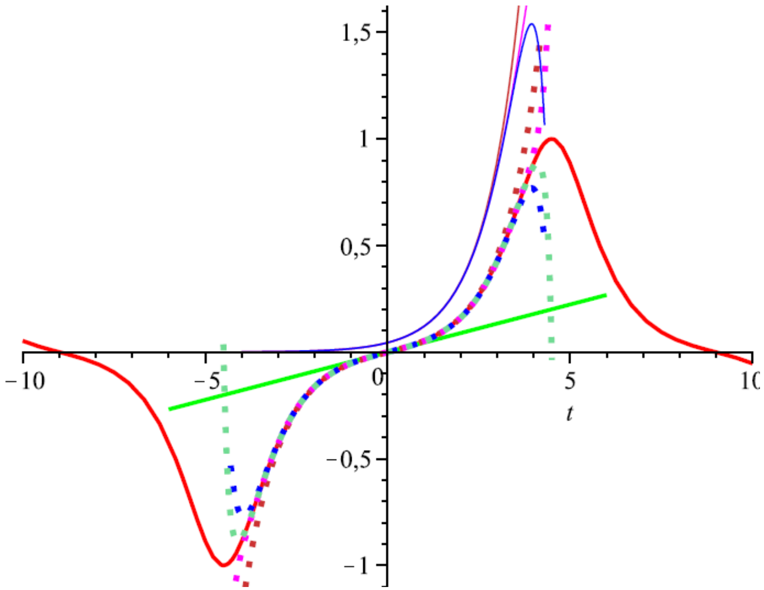
Next 3 transparencies an explanation of the origin of these formula

Nonlinear tachyon

$$\ddot{q} - \mu^2 q = -\epsilon\, q^3, \quad \epsilon > 0,$$
$$I.D.: \quad q(0) = q_0, \quad \dot{q}(0) = v_0$$

$$E = \frac{1}{2}v_0^2 - \frac{1}{2}\mu^2 q_0^2 + \frac{1}{4}\epsilon q_0^4$$

$E > 0$ — motion in two holes



$$q(t) = a\,\mathrm{cn}(\Omega t + b, k)$$
$$a^2 = \frac{\mu^2}{\epsilon}\left(1 + \sqrt{1 + \frac{4\epsilon E}{\mu^4}}\right)$$
$$\Omega^2 = \mu^2\sqrt{1 + \frac{4\epsilon E}{\mu^4}}$$
$$k^2 = \frac{1}{2} + \frac{1}{2}\frac{1}{\sqrt{1 + \frac{4\epsilon E}{\mu^4}}}$$
$$b \;:\; \text{from } q_0 = a\,\mathrm{cn}(b, k)$$

Nonlinear tachyon

$$q(u) = \mathrm{cn}(u - \mathbf{K}, k) \tag{30}$$

Asymptotic expansion for $|u| < \mathbf{K}$ and small k'

$$\mathrm{cn}(u - \mathbf{K}, k) = \frac{\mathrm{sn}(u,k)}{\mathrm{dn}(u,k)} k' = \frac{\pi}{k\mathbf{K}'} \left\{ \frac{\sinh u'}{\cosh \rho'} - \frac{\sinh 3u'}{\cosh 3\rho'} + \frac{\sinh 5\rho'}{\cosh 5u'} + \dots \right\} \tag{31}$$

where

$$\rho' = \frac{\pi \mathbf{K}}{2\mathbf{K}'}, \quad u' = \frac{\pi u}{2\mathbf{K}'} \tag{32}$$

$$\frac{1}{\cosh \rho'} = \frac{2}{e^{\rho'} + e^{-\rho'}} = \frac{2}{\frac{1}{\mathrm{q}'^{1/2}} + \mathrm{q}'^{1/2}} = \frac{2\sqrt{\mathrm{q}'}}{1 + \mathrm{q}'}, \quad \mathrm{q}' = e^{-\frac{\pi \mathbf{K}}{\mathbf{K}'}}, \tag{33}$$

These series converge for $|\Re u| < \mathbf{K}$ and $-1 < k < 1$. Expansion of $1/\cosh \rho'$ for small k' is

$$\frac{1}{\cosh \rho'} = \frac{1}{2} k' + \frac{3}{32} k'^3 + \mathcal{O}(k'^5) \tag{34}$$

I.A.,E.Piskowsky,I.Volovich

Stretch of the kinetic energy

- Stretch of the kinetic energy

$$e^{\tau(\partial^2+3H\partial)}(\partial^2+3H\partial-\mu^2)\phi(t)=-\epsilon\phi^3(t)$$

Generalization of
Lidsey, Int.J.Mod.Phys,2008
Barnaby, Cline, 0802.3218

- Approximation

$$\ddot{\phi}+3\sqrt{\frac{8\pi G}{3}\left(e^{\tau\Omega^2}\left(\frac{1}{2}\dot{\phi}^2-\frac{\mu^2}{2}\phi^2\right)+\frac{\epsilon}{4}\phi^4+\Lambda\right)}\,\dot{\phi}=\mu^2\phi-\epsilon\phi^3$$

Consequences of the Stretch of the Kinetic Energy

- **Appearance of forbidden region**

$$\ddot{\phi} + 3\sqrt{\frac{8\pi G}{3}\left(e^{\tau\Omega^2}\left(\frac{1}{2}\dot{\phi}^2 - \frac{\mu^2}{2}\phi^2\right) + \frac{\epsilon}{4}\phi^4 + \Lambda\right)}\,\dot{\phi} = \mu^2\phi - \epsilon\phi^3$$

Assume Higgs potential

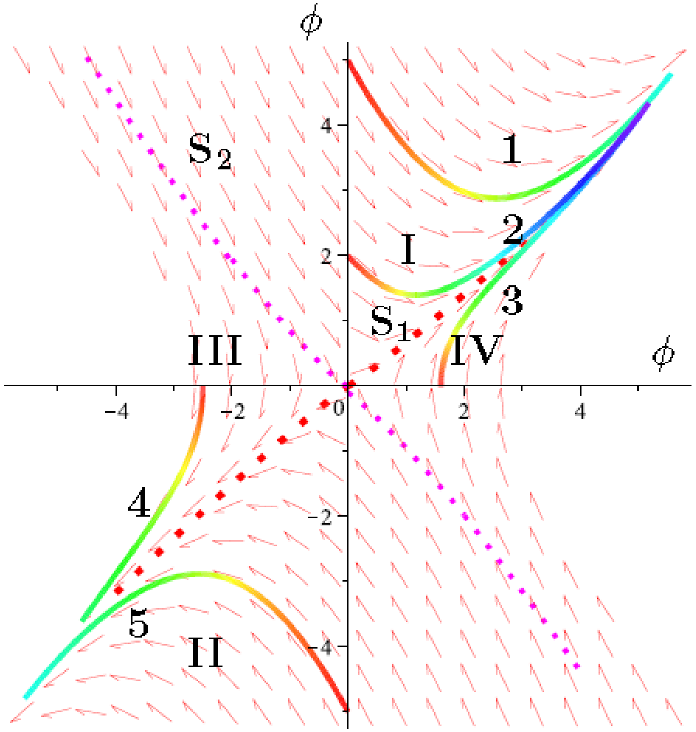
$$-\frac{\mu^2}{2}\phi^2 + \frac{\epsilon}{4}\phi^4 + \Lambda = \frac{\epsilon}{4}(\phi^2 - \phi_0^2)^2$$

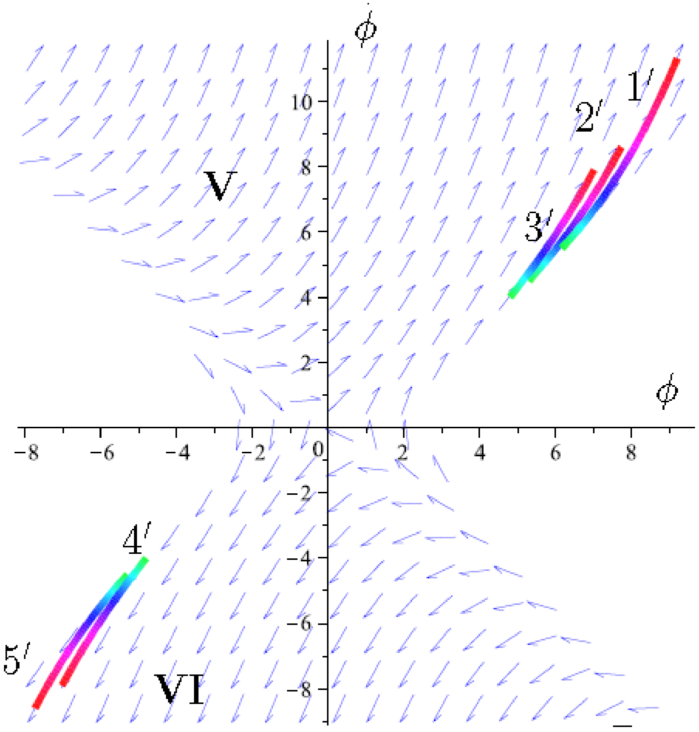
$$\mu^2 = \epsilon\phi_0^2, \quad \Lambda = \frac{\epsilon\phi_0^4}{4}$$

$$\left(-\frac{\mu^2}{2}e^{\tau\Omega^2}\phi^2 + \frac{\epsilon}{4}\phi^4 + \Lambda\right) = \frac{\epsilon}{4}(\phi^2 - \phi_0^2)^2 - \frac{\mu^2}{2}\left(e^{\tau\Omega^2} - 1\right)$$

Inflation with Tachyon?

$$\ddot{\phi} + 3\sqrt{\frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 - \frac{\mu^2}{2}\phi^2 + \Lambda\right)}\,\dot{\phi} = \mu^2\phi$$



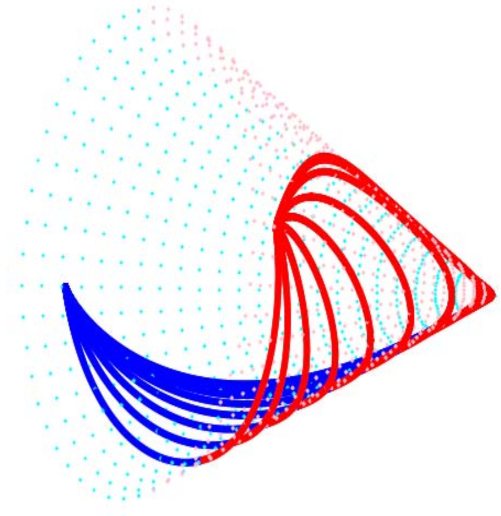


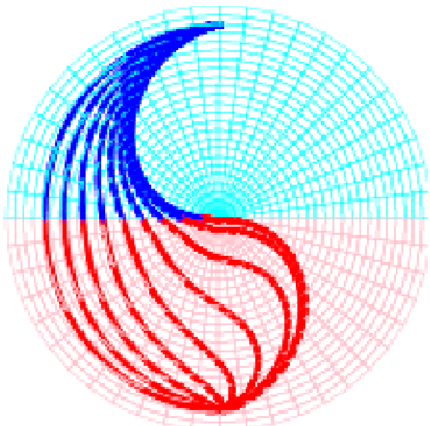
Inflation with Tachyon?

$$\ddot{\phi} + 3\sqrt{\frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 - \frac{\mu^2}{2}\phi^2 + \Lambda\right)}\,\dot{\phi} = \mu^2\phi$$

$\Lambda = 0$

Dynamical system with 3 DOFs in the unit ball





$$\phi = const - \frac{1}{\sqrt{12\pi G}}\ln t, \quad H \simeq \frac{1}{3t}, \qquad a \sim t^{1/3}$$

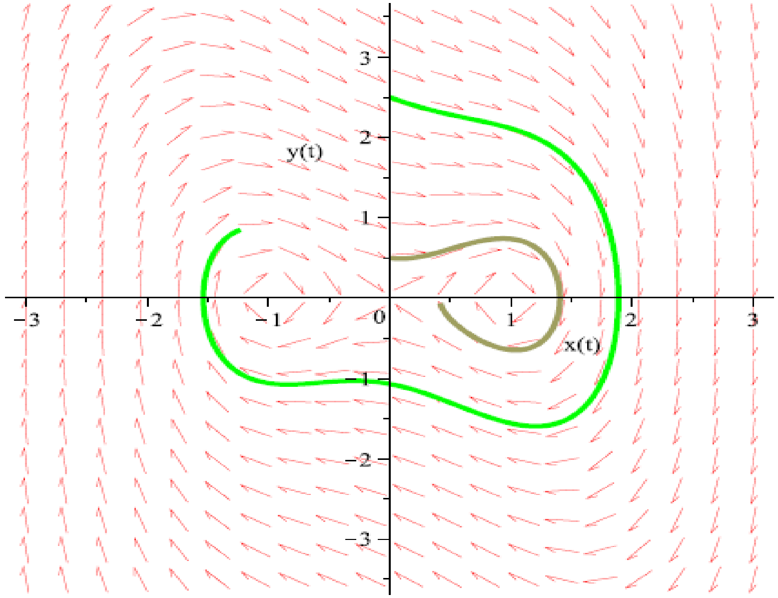
$$\phi = const + \frac{1}{\sqrt{12\pi G}}\ln t, \quad H \simeq -\frac{1}{3t}, \qquad a \sim -t^{1/3}$$

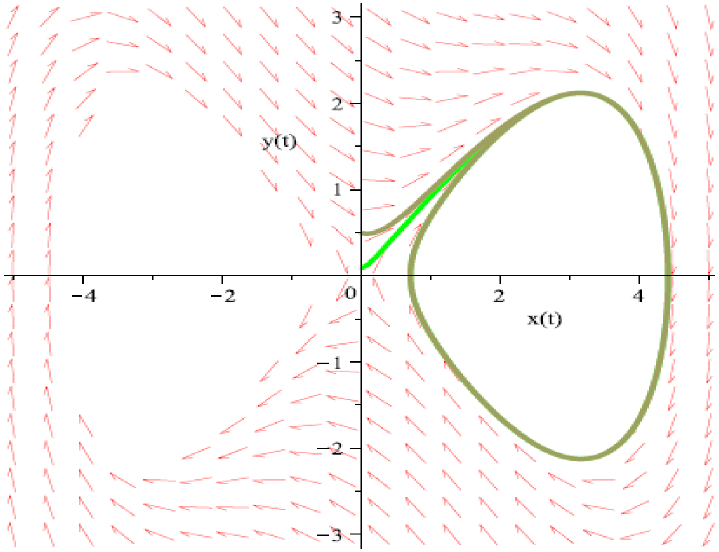
Inflation with Higgs potential

$$\ddot{\phi} + 3\sqrt{\frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{\epsilon}{4}\phi^4 - \frac{\mu^2}{2}\phi^2 + \Lambda\right)}\,\dot{\phi} = \mu^2\phi - \epsilon\phi^3$$

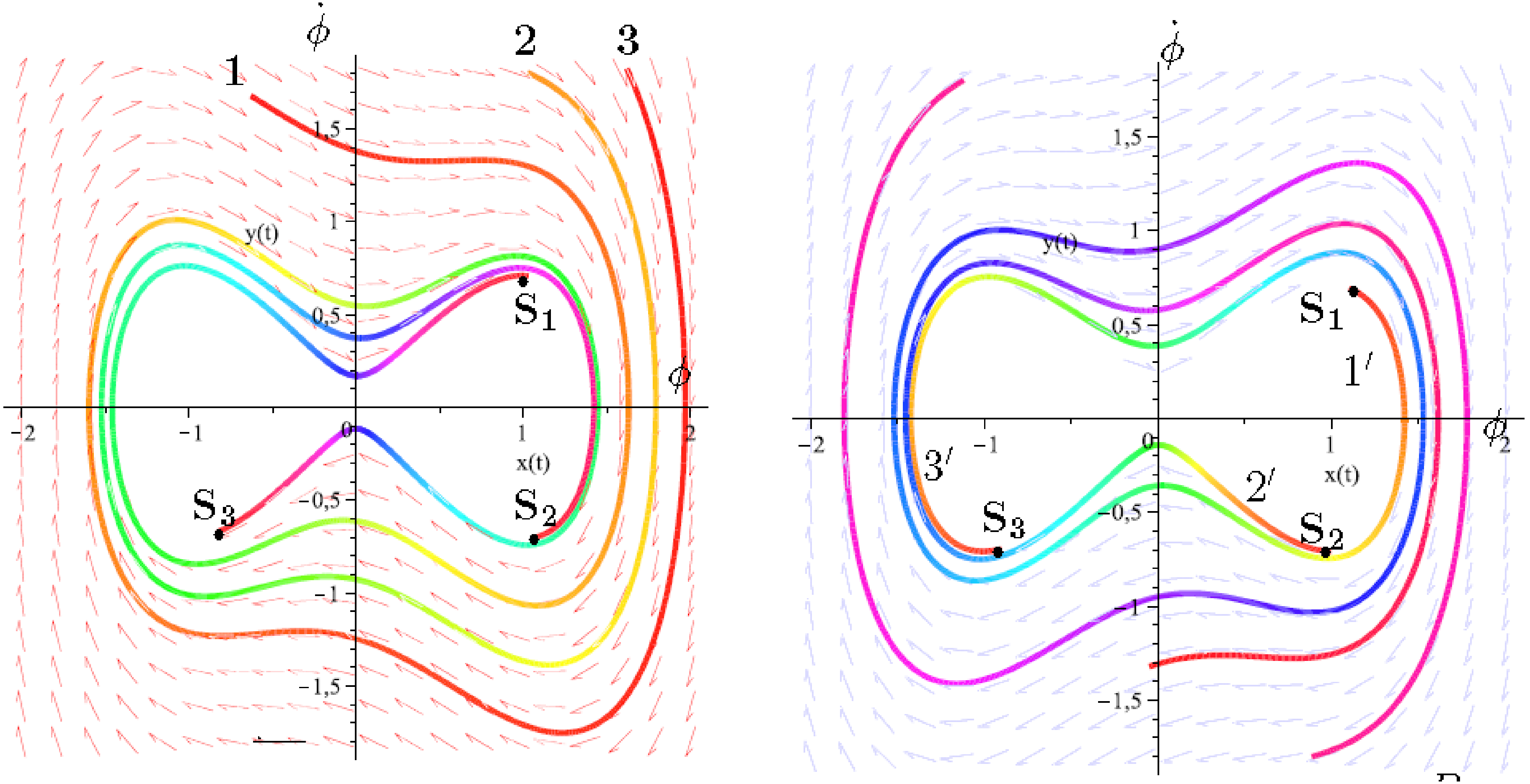
2 different situations. The potential energy is:

positive defined **not positive defined**



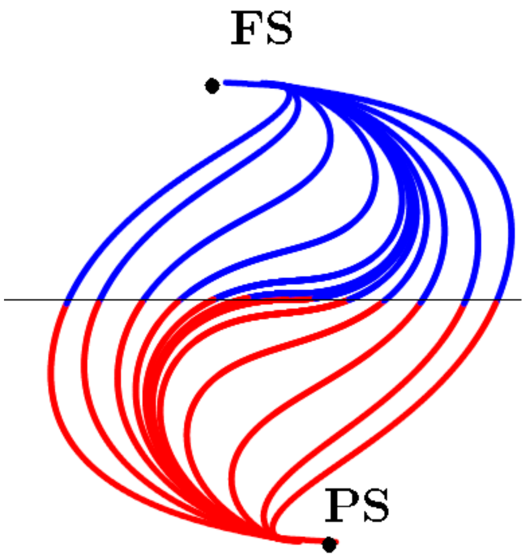


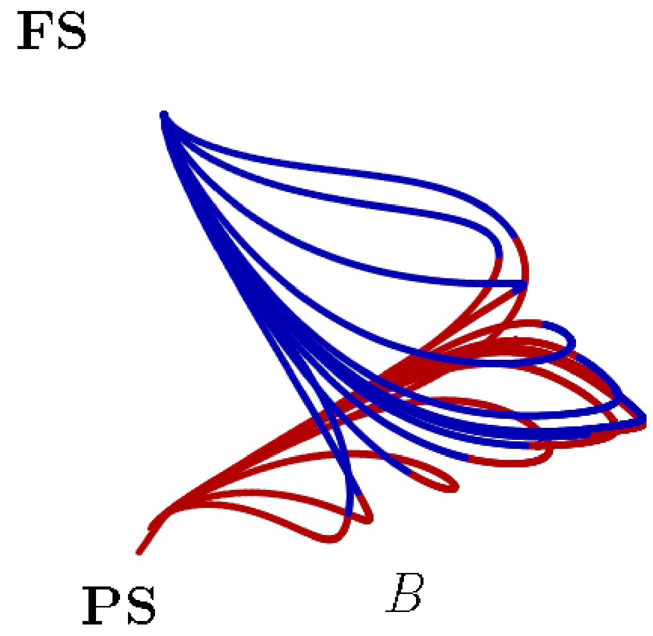
Higgs with a negative cosmological constant



Higgs with a negative cosmological constant

$$\ddot{\phi} + 3\sqrt{\frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{\epsilon}{4}\phi^4 - \frac{\mu^2}{2}\phi^2 + \Lambda\right)}\,\dot{\phi} = \mu^2\phi - \epsilon\phi^3$$





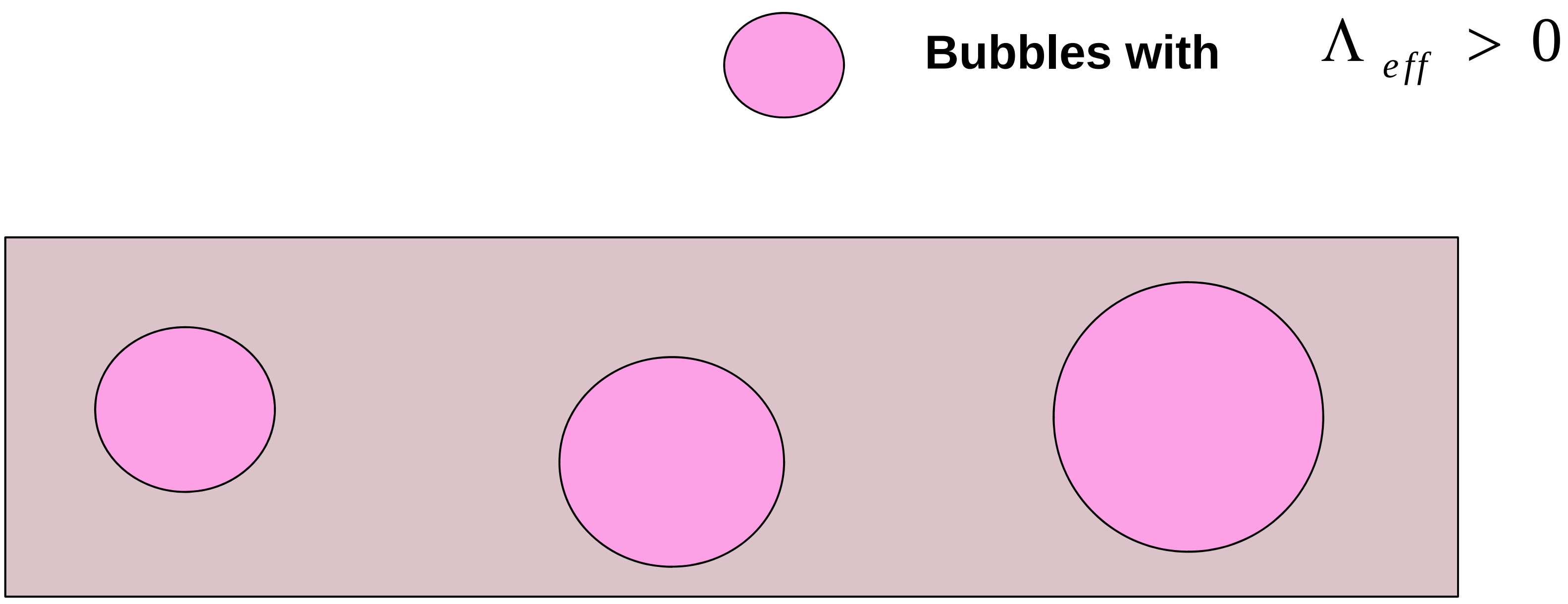
Nonlocal (SFT-inspired) inflation

- An extra current

- Effective negative cosmological constant

Anthropic Principle

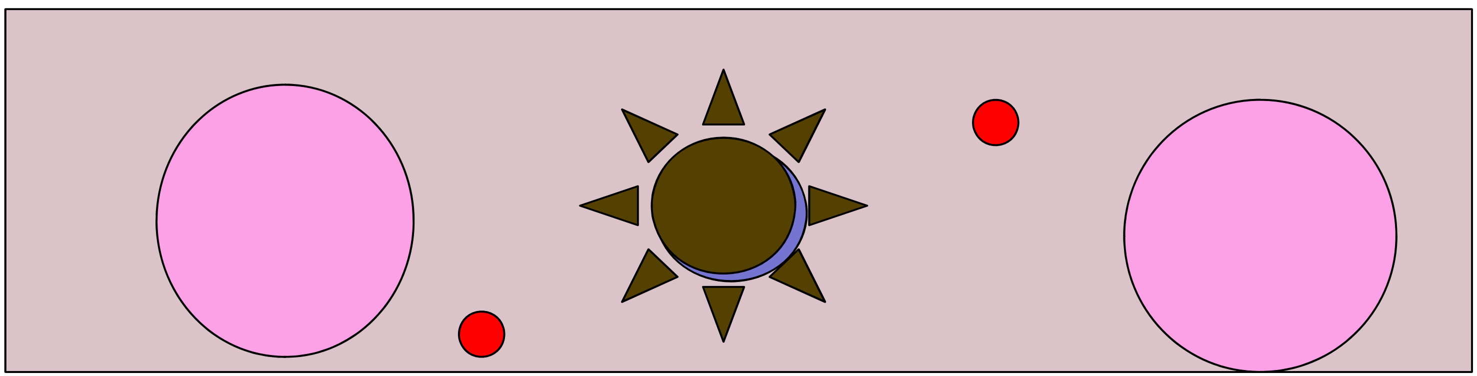
- **The anthropic principle: there are many universes**



Effective negative cosmological constant

Bubbles with $\Lambda_{eff} < 0$

Bubbles with $\Lambda_{eff} > 0$



Predators

Escape

Gravitational collapse

Few Refs on SFT Inspired Nonlocal Models in Cosmology

- **Later cosmology**
  w<-1

- I.A., astro-ph/0410443
  I.A., L.Joukovkaya, JHEP,05109 (2005) 087
  I.A., A.Koshelev, JHEP, 07022 (2007) 041
  L.Joukovskaya, PR D76 (2007) 105007;
  JHEP (2009)
  G. Calcagni, M.Montobbio,G.Nardelli,
  0705.3043; 0712.2237;
  Calcagni, Nardelli, 0904.424 ……..

- **Inflation**
  steep potential,
  non-gaussianity

- N. Barnaby, T. Biswas, J.M. Cline, hep-th/0612230,
  J.Lidsey, hep-th/0703007;
  Nunes, Mulryne, 0810.5471;
  I. A., I.Volovich, arXiv: 1103.0273,

- **Bouncing solutions**

I.A., L.Joukovskaya, S.Vernov, JHEP 0707 (2007) 087
…….