The BV Formulation of Non-Polynomial NS String Field Theory

Work in progress w. ***N. Berkovits, Y. Okawa, M. Schnabl, S. Torii and B. Zwiebach***

Michael Kroyter
Marie Curie IOF Fellow – Tel-Aviv University

SFT 2011 – Prague
23-Sep-2011

Outline

1. Introduction
2. The gauge structure of SSFT
3. Perturbative approach
4. Non-perturbative methods
5. Summary and outlook

Outline

1. Introduction
2. The gauge structure of SSFT
3. Perturbative approach
4. Non-perturbative methods
5. Summary and outlook

Motivation

- Gauge symmetry must be carefully taken care of lest a wrong count of d.o.f. lead to erroneous results regarding observables.
- A very useful method for handling gauge symmetry is the ***BRST*** approach, in which the space of classical observables is identified with the cohomology of the BRST operator.
- The usual BRST construction is Hamiltonian in nature and is therefore not manifestly covariant.
- A Lagrangian covariant BRST formalism exists and goes under the name of the ***BV*** (Batalin-Vilkovisky) or field-antifield approach.
- The BV formalism is particularly useful when the gauge symmetry is reducible / uses the equations of motion.

Motivation

- Gauge symmetry must be carefully taken care of lest a wrong count of d.o.f. lead to erroneous results regarding observables.
- A very useful method for handling gauge symmetry is the ***BRST*** approach, in which the space of classical observables is identified with the cohomology of the BRST operator.
- The usual BRST construction is Hamiltonian in nature and is therefore not manifestly covariant.
- A Lagrangian covariant BRST formalism exists and goes under the name of the ***BV*** (Batalin-Vilkovisky) or field-antifield approach.
- The BV formalism is particularly useful when the gauge symmetry is reducible / uses the equations of motion.
- ***String field theory*** is formulated using a Lagrangian approach and its gauge structure is complicated $\Longrightarrow$ BV approach is advisable.
- The BV treatment of the bosonic theories is known. What about the ***superstring***? Particularly, the non-polynomial NS SFT (Berkovits 95).

The BV formalism

- Assume that an action S_0 depending on some fields ϕ_α has a gauge symmetry: $\delta\phi_\alpha = R_\alpha^\beta \epsilon_\beta$ and presumably also gauge for gauge...
- Introduce ghosts $\phi_\beta^{(1)}$, ghosts for ghosts $\phi_\gamma^{(2)}$ and so on, as in the usual BRST formalism.
- Introduce antifields ϕ^{I*} for the fields (including ghosts).
- Define the ***antibrackets*** as an odd symplectic structure with respect to all the pairs of fields (including ghosts) and antifields.
- The action is extended to include ghosts and antifields. This ***Master Action*** generates BRST transformations via the antibrackets:

$$\delta_B A = \{A, S\}.$$

- Nilpotency of the BRST operator and BRST invariance of the action are equivalent to the ***Classical Master Equation:*** $\{S, S\} = 0$.
- The action should also obey the ***initial conditions:***

$$S|_{\phi^{I*}=0} = S_0\,, \qquad \frac{\delta^L}{\delta\phi^{(n)\sigma *}} \frac{\delta^R}{\delta\phi_\rho^{(n+1)}} S\Big|_{\phi^{I*}=0} = R_\sigma^{(n)\rho}\,.$$

BV for string field theory

Master equation for ***string fields***?

If everything depends on string fields, we can write the master equation in terms of component fields and re-express it in terms of string fields.

Example: Bosonic open SFT (Bochicchio 87, Thorn 87)

- ***Infinitely reducible*** gauge symmetry exists ***on-shell***:
  $\delta\Psi_1 = Q\Lambda_0 + [\Psi_1, \Lambda_0] \equiv \mathcal{Q}\Lambda_0 \qquad \delta\Lambda_n = \mathcal{Q}\Lambda_{n-1}$
- Infinitely many ghost string fields: $\Psi_n \quad n \leq 0$. All are odd.
- Infinitely many string antifields: $\Psi_n \quad n \geq 2$. All are odd.
- In terms of the string fields the anti-brackets are:
  $$\{A, B\} = \sum_{g=-\infty}^{\infty} \int \frac{\delta_R A}{\delta \Psi_g} \frac{\delta_L B}{\delta \Psi_{3-g}} \qquad \delta A = \int \frac{\delta_R A}{\delta \Psi_g} \delta \Psi_g$$
- For the master string field: $\Psi = \sum_{g=-\infty}^{\infty} \Psi_g\,, \qquad \{A, B\} = \int \frac{\delta_R A}{\delta \Psi} \frac{\delta_L B}{\delta \Psi}$
- The master action $S \equiv -\int \left(\frac{1}{2}\Psi Q \Psi + \frac{1}{3}\Psi^3\right)$ solves the master equation and obeys the boundary conditions.

Outline

1 Introduction

2 The gauge structure of SSFT

3 Perturbative approach

4 Non-perturbative methods

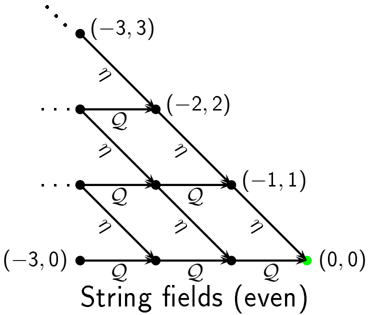
5 Summary and outlook

The gauge structure of SSFT

$$\mathcal{Q}X \equiv QX + [e^{-\phi}Qe^{\phi}, X]$$

$$\begin{aligned}
e^{-\phi}\delta e^{\phi} &= \mathcal{Q}\epsilon_{-1,0} + \eta\epsilon_{-1,1} \\
\delta\epsilon_{-1,0} &= \mathcal{Q}\epsilon_{-2,0} + \eta\epsilon_{-2,1} \\
\delta\epsilon_{-2,1} &= \mathcal{Q}\epsilon_{-3,1} + \eta\epsilon_{-3,2} \\
&\vdots
\end{aligned}$$

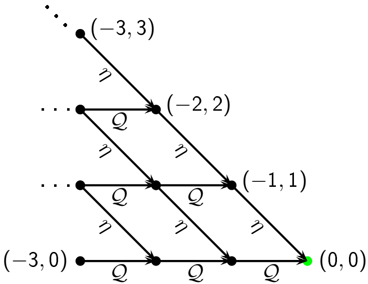
The gauge structure of SSFT



$$\mathcal{Q}X \equiv QX + [e^{-\phi}Qe^{\phi}, X]$$

$$e^{-\phi}\delta e^{\phi} = \mathcal{Q}\epsilon_{-1,0} + \eta\epsilon_{-1,1}$$
$$\delta\epsilon_{-1,0} = \mathcal{Q}\epsilon_{-2,0} + \eta\epsilon_{-2,1}$$
$$\delta\epsilon_{-2,1} = \mathcal{Q}\epsilon_{-3,1} + \eta\epsilon_{-3,2}$$
$$\vdots$$

The gauge structure of SSFT

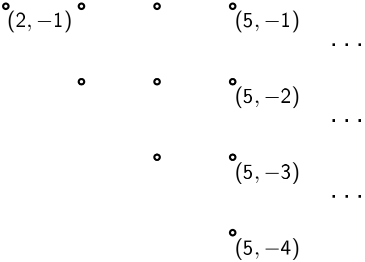


String fields (even)

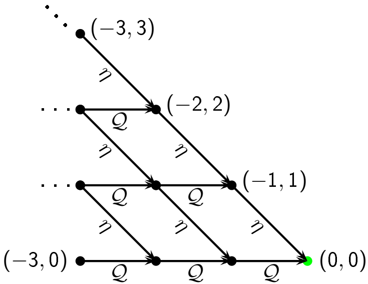
$$\mathcal{Q}X \equiv QX + [e^{-\phi}Qe^{\phi}, X]$$

$$e^{-\phi}\delta e^{\phi} = \mathcal{Q}\epsilon_{-1,0} + \eta\epsilon_{-1,1}$$
$$\delta\epsilon_{-1,0} = \mathcal{Q}\epsilon_{-2,0} + \eta\epsilon_{-2,1}$$
$$\delta\epsilon_{-2,1} = \mathcal{Q}\epsilon_{-3,1} + \eta\epsilon_{-3,2}$$
$$\vdots$$

String antifields (odd)



The gauge structure of SSFT

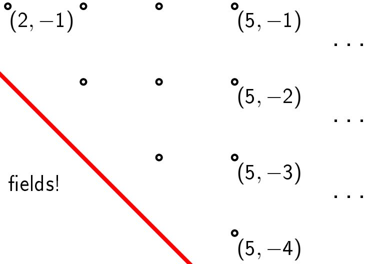


String fields (even)

$$\mathcal{Q}X \equiv QX + [e^{-\phi}Qe^{\phi}, X]$$

$$e^{-\phi}\delta e^{\phi} = \mathcal{Q}\epsilon_{-1,0} + \eta\epsilon_{-1,1}$$
$$\delta\epsilon_{-1,0} = \mathcal{Q}\epsilon_{-2,0} + \eta\epsilon_{-2,1}$$
$$\delta\epsilon_{-2,1} = \mathcal{Q}\epsilon_{-3,1} + \eta\epsilon_{-3,2}$$
$$\vdots$$

String antifields (odd)



$$S = S_0 + \oint \Big(\Phi_{2,-1}e^{\phi}\left(\mathcal{Q}\Phi_{-1,0} + \eta\Phi_{-1,1}\right) + \Phi_{3,-1}\left(\mathcal{Q}\Phi_{-2,0} + \eta\Phi_{-2,1}\right) + \Phi_{3,-2}\left(\mathcal{Q}\Phi_{-2,1} + \eta\Phi_{-2,2}\right) + \dots\Big)$$

Cannot add fields and antifields. Cannot even sum up all fields!

$$\{A, B\} = \sum_{I\in\Delta_-} \oint \left(\frac{\delta_R A}{\delta\Phi_I}\frac{\delta_L B}{\delta\Phi^{I*}} - \frac{\delta_R A}{\delta\Phi^{I*}}\frac{\delta_L B}{\delta\Phi_I}\right)$$

Outline

1. Introduction
2. The gauge structure of SSFT
3. Perturbative approach
4. Non-perturbative methods
5. Summary and outlook

Perturbative approach

Theorem: solving the master equation order by order in antifield number leads to a solution, i.e., all solutions for the master action at a particular antifield number can be extended to full solutions.

The antifield $\Phi_{2,-1}$ of the classical field e^ϕ carries antifield number one. $\Phi_{3,-1}$ and $\Phi_{3,-2}$ have antifield number two...

The action should be expanded as: $S = S_0 + S_1 + \ldots$, with S_0 the classical action, $S_1 = \oint \Phi_{2,-1} e^\phi (\mathcal{Q}\Phi_{-1,0} + \eta\Phi_{-1,1})$ and so on.
S_2 and higher terms might include ***non-initial-condition terms***.

The master equation is:

$$\oint \left(\frac{\delta_R S}{\delta e^\phi} \frac{\delta_L S}{\delta \Phi_{2,-1}} + \left(\frac{\delta_R S}{\delta \Phi_{-1,0}} \frac{\delta_L S}{\delta \Phi_{3,-1}} + \frac{\delta_R S}{\delta \Phi_{-1,1}} \frac{\delta_L S}{\delta \Phi_{3,-2}} \right) + \ldots \right) = 0$$

The 1^{st} term reduces antifield number by one, the next ones by two, etc...

Perturbative approach – lowest orders

The lowest order equation is: $\oint \frac{\delta_R S_0}{\delta e^\phi}\frac{\delta_L S_1}{\delta \Phi_{2,-1}} = 0$. There is no freedom of choosing a solution. It is reassuring to verify that the equation holds.

The next order is already non-trivial:

$$\oint \Big(\frac{\delta_R S_0}{\delta e^\phi}\frac{\delta_L S_2}{\delta \Phi_{2,-1}} + \frac{\delta_R S_1}{\delta e^\phi}\frac{\delta_L S_1}{\delta \Phi_{2,-1}} + \frac{\delta_R S_1}{\delta \Phi_{-1,0}}\frac{\delta_L S_2}{\delta \Phi_{3,-1}} + \frac{\delta_R S_1}{\delta \Phi_{-1,1}}\frac{\delta_L S_2}{\delta \Phi_{3,-2}}\Big) = 0$$

Solutions are obtained by adding non-trivial terms to S_2. For example:

$$\delta S_2 = \oint \Big(\Phi_{2,-1} e^\phi \Phi_{2,-1} e^\phi \Phi_{-2,1} + \Phi_{3,-1}\big(\Phi_{-1,0}\mathcal{Q}\Phi_{-1,0} - [\eta\Phi_{-1,1}, \Phi_{-1,0}]\big) + \Phi_{3,-2}\Phi_{-1,1}\eta\Phi_{-1,1}\Big)$$

Note that:

- The blue non-derivative term is uniquely fixed.
- The solution is a specific case of a four-parameter family.
- e^ϕ only appears in $\mathcal{Q}$ and in the combination $\Phi_{2,-1}e^\phi$.

Perturbative approach – higher orders and $\mathbb{Z}_2$ symmetry

Problems

- The solution for δS_3 already contains 20 new free parameters.
- Within the 24-dimensional family of solutions for S_2 and S_3 we do not see any specific one that has a natural extension to higher orders.
- Many more terms at higher orders, including quartic and quintic terms: How should we proceed?

Possible strategy: Enforce manifest $\mathbb{Z}_2$ symmetry

The $\mathbb{Z}_2$ symmetry of the large Hilbert space interchanges Q and η.
It also sends ϕ to $-\phi$. It can be extended to all the other fields.
Enforcing it on the solutions reduces the amount of free parameters by a factor of two. However:

- Still many free parameters.
- Still no "natural" solution.
- Not clear whether the simplest solution should be a $\mathbb{Z}_2$-invariant one.

Field redefinition

In the bosonic case the solution took a very simple form when expressed in terms of the master string field.
Could we define $\Phi = \sum_{(g,p)\in\Delta_-} \Phi_{g,p} \qquad \Phi^* = \sum_{(g,p)\in\Delta_+} \Phi_{g,p}$?
Either treat e^ϕ differently or redefine the fields in order to get a homogeneous transformation law ($V \equiv e^{-\phi} Q e^{\phi}\,, \quad V_\eta \equiv e^{\phi}\eta e^{-\phi}$):

$$\begin{aligned}
\delta e^{\phi} &= Q\epsilon_{-1,0} + \epsilon_{-1,0} V + \eta\epsilon_{-1,1} + V_\eta \epsilon_{-1,1} \\
\delta \epsilon_{-1,0} &= Q\epsilon_{-2,0} - \epsilon_{-2,0} V + \eta\epsilon_{-2,1} + V_\eta \epsilon_{-2,1} \\
\delta \epsilon_{-1,1} &= Q\epsilon_{-2,1} - \epsilon_{-2,1} V + \eta\epsilon_{-2,2} + V_\eta \epsilon_{-2,2} \\
&\vdots
\end{aligned}$$

Still, no solution to the master equation in terms of Φ and Φ^*:

- Transformation law depends on e^ϕ.
- Boundaries.

ϕ as the original field

Maybe we get complicated expression since we work with the wrong variables. Let ϕ be the classical field instead of e^{ϕ}.
Problem: What is the gauge transformation of ϕ?

This is simply written using L_ϕ, the adjoin operation of ϕ: $L_\phi A \equiv [\phi, A]$
The classical action is (manifestly 2d rep. for WZW): $S_0 = \oint J_\eta e^{-L_\phi} J_Q$
(J_Q, J_η are integrals of the BRST/eta current over the identity)

The gauge symmetry is: $\delta\phi = \frac{\frac{1}{2}L_\phi}{\sinh\left(\frac{1}{2}L_\phi\right)}\left(e^{-\frac{1}{2}L_\phi}Q\epsilon_{-1,0} + e^{\frac{1}{2}L_\phi}\eta\epsilon_{-1,1}\right)$
Manifestly $\mathbb{Z}_2$ invariant (note: $\mathcal{Q} = e^{-L_\phi}Qe^{L_\phi}$)

We constructed the first few terms of the solution.
Still, not clear how does the full solution looks like.

Outline

1. Introduction
2. The gauge structure of SSFT
3. Perturbative approach
4. Non-perturbative methods
5. Summary and outlook

Canonical transformations

As in Hamiltonian dynamics, a possible way towards the identification of the correct variables is the use of a canonical transformation.
In particular, the two previous field redefinitions ("point transformations") can be realised as canonical transformations.

Canonical transformations do not change the (anti)bracket structure.
How to realise if terms of ***string fields***?

Canonical transformations

As in Hamiltonian dynamics, a possible way towards the identification of the correct variables is the use of a canonical transformation.
In particular, the two previous field redefinitions ("point transformations") can be realised as canonical transformations.

Canonical transformations do not change the (anti)bracket structure.
How to realise if terms of ***string fields***?

Let f, g be two arbitrary test string fields that do not depend on the canonical string fields:

$$\Big\{ \oint f\Phi_I, \oint \Phi^{J*} g \Big\} = \Big\{ \oint f\tilde{\Phi}_I, \oint \tilde{\Phi}^{J*} g \Big\} = \delta_I^J \oint fg$$

$$\Big\{ \oint f\Phi_I, \oint \Phi_J g \Big\} = \Big\{ \oint f\tilde{\Phi}_I, \oint \tilde{\Phi}_J g \Big\} = 0$$

$$\Big\{ \oint f\Phi^{I*}, \oint \Phi^{J*} g \Big\} = \Big\{ \oint f\tilde{\Phi}^{I*}, \oint \tilde{\Phi}^{J*} g \Big\} = 0$$

Found explicit canonical transformations among the various coordinate systems. Gained some intuition.

Using gradings to restrict the action

Several additive quantum numbers can be assigned to terms in the action:

- g: World-sheet ghost number. Adds up to 2 (large Hilbert space).
- p: Picture number. Adds up to -1.
- G: BV total ghost number. Equals BV ghost number minus antifield number. Adds up to 0.
- N_Q: Number of Q's.
- N_η: Number of η's.
- N_A: Number of antifields.

Using gradings to restrict the action

Several additive quantum numbers can be assigned to terms in the action:

- g: World-sheet ghost number. Adds up to 2 (large Hilbert space).
- p: Picture number. Adds up to -1.
- G: BV total ghost number. Equals BV ghost number minus antifield number. Adds up to 0.
- N_Q: Number of Q's.
- N_η: Number of η's.
- N_A: Number of antifields.

For the string fields: $G=-g$, while for the antifields: $G=1-g$.
Q and η contribute to g, but not to G.
Thus: $G=-g+N_A+N_Q+N_\eta \quad\Longrightarrow\quad N_A+N_Q+N_\eta=2$
N_A, N_Q, N_η are all non-negative. Three possible types of terms:

- $N_A=0 \quad\Longrightarrow\quad$ only ϕ appears. This is S_0.
- $N_A=1 \quad\Longrightarrow\quad$ either N_Q or N_η equals one.
- $N_A=2 \quad\Longrightarrow\quad$ non-derivative terms.

The result is consistent with parity.

An aside: Grading for the bosonic/modified theories

- g: World-sheet ghost number. Adds up to 3 (bosonic/small space).
- G: BV total ghost number. Adds up to 0.
- N_Q: Number of Q's.
- N: Total number of fields (including antifields).

Now, for all types of fields: $G = 1 - g$
Thus: $G = N - g + N_Q \quad \Longrightarrow \quad N + N_Q = 3$
That is, the only possible terms look like $\Psi Q \Psi$ and Ψ^3

Remarks:

- We knew that already.
- One could have presumably devised a canonical transformation that does not respect g and add higher order terms.
- Very similar result holds for the democratic theory with a slightly different derivation.

Further use of the gradings

We can now decompose the action as: $S = S^{1,1} + S^{1,0} + S^{0,1} + S^{0,0}$.
The two indices represent the number of Q's and η's respectively.
Note: $S^{1,1} = S_0$, each term of $S^{1,0}$, $S^{0,1}$ includes exactly one antifield and $S^{0,0}$ terms include two, $S^{1,0}$ and $S^{0,1}$ are $\mathbb{Z}_2$ dual, the others are self-dual.

Decomposing the master equation according to the number of Q's and η's shows, e.g., that $S^{0,0}$ itself obeys the master equation.

Further use of the gradings

We can now decompose the action as: $S = S^{1,1} + S^{1,0} + S^{0,1} + S^{0,0}$.
The two indices represent the number of Q's and η's respectively.
Note: $S^{1,1} = S_0$, each term of $S^{1,0}$, $S^{0,1}$ includes exactly one antifield and $S^{0,0}$ terms include two, $S^{1,0}$ and $S^{0,1}$ are $\mathbb{Z}_2$ dual, the others are self-dual.

Decomposing the master equation according to the number of Q's and η's shows, e.g., that $S^{0,0}$ itself obeys the master equation.

Further grading: N_G number of ghost fields (including neither antifields nor the classical field).

Decomposing the master equation again leads to a simple equation for the component of $S^{0,0}$ at $N_G = 1$. The solution can be written in terms of master fields: $\Phi \equiv \sum_{g<0,p} \Phi_{g,p} \qquad \Phi_{2,-1} e^{\phi} + \sum_{g>2,p} \Phi_{g,p}$

$$S^{0,0}\big|_{N_G=1} = \oint \Phi^{*2} \Phi$$

Outline

1. Introduction
2. The gauge structure of SSFT
3. Perturbative approach
4. Non-perturbative methods
5. Summary and outlook

Summary and outlook

Summary

- Identified the gauge structure of the theory
- Wrote the initial conditions terms
- Evaluated some terms perturbatively
- Found restrictions on the form of possible solution
- Evaluated some terms non-perturbatively
- ***Still no closed form expression for the solution***

Summary and outlook

Summary

- Identified the gauge structure of the theory
- Wrote the initial conditions terms
- Evaluated some terms perturbatively
- Found restrictions on the form of possible solution
- Evaluated some terms non-perturbatively
- ***Still no closed form expression for the solution***

Further directions

- Finding the correct variables
- Non-minimal sectors
- Deriving from the BV of a cubic theory (modified/Grassi-Schnabl/democratic)

Summary and outlook

Summary

- Identified the gauge structure of the theory
- Wrote the initial conditions terms
- Evaluated some terms perturbatively
- Found restrictions on the form of possible solution
- Evaluated some terms non-perturbatively
- ***Still no closed form expression for the solution***

Further directions

- Finding the correct variables
- Non-minimal sectors
- Deriving from the BV of a cubic theory (modified/Grassi-Schnabl/democratic)
- ***Are we solving the wrong problem?***