Lattice String Field Theory
Work in progress w. ***Francis Bursa*** (Cambridge)

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Outline

1. Introduction
2. Obstacles towards a lattice approach and some resolutions
3. The SFT side
4. The lattice side
5. Conclusions and future directions

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- Could open SFT be consistent by itself/capture the closed string physics? Should a closed SFT exist?
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- Does the RNS string possess a well defined measure?
- Could we study "the landscape" using SFT methods?

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How could we hope to answer these questions?
Analytical study of non-perturbative, quantum SFT is hard.
Why not try an "experimental" lattice approach?

For many reasons the idea is ***crazy***.
If successful, the reward might be ***very high***.

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General obstacles

SFT side

- Which string field theory to use? (each one has its own problems)
- Gauge freedom: Should we fix it? BRST/BV?
- Infinite number of fields: Level truncation breaks gauge/BRST invariance. It also introduces an additional cut-off.

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Lattice side

- The theories are consistent at dimensions that are impossible for lattice simulations: $3^{26} > 1200^4$.
- Tensors of arbitrarily high rank:
  Where to put them (links/plaquettes)?
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And more...

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- No Poincaré-invariance: Inequivalent integration regions.
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Possible resolutions

- Introduce a cosmological constant term:
  Have to solve the world-sheet theory first.
- Hope that an “order of limits” would lead to reliable results.

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- Use analytical continuation: $\Psi \star \Psi \star \Psi \to i\Psi \star \Psi \star \Psi$.
  - ***Do we get real values for observables (e.g. action)?***
    We examine this question as a test for the viability of the approach.

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The string field as the basic entity

We might have a chance to make the lattice theory work if we treat the string field as a single entity.

- All component fields, regardless of spin, live on lattice sites.
- There is a single truncation for the whole string field.

We consider the string field in the range $x_{min} < x < x_{max}$.
Eventually $x_{min} \to -\infty$ and $x_{max} \to \infty$.

The string field is then $\Psi = \big(T(x) + a_{-1}A(x) + ...\big)c_1 |0\rangle$.
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How to combine with level truncation?

Level truncation

- The level is defined in momentum space as $l = l_0 + \alpha' p^2$, with $l_0(T) = 0$, $l_0(A) = 1 \ldots$ being the "usual" levels.
  The second term gives the spatial dependence.
  l as the level was considered in the past in the context of lump solutions (Moeller, Sen and Zwiebach 00).
- We expand the fields in momentum, while increasing l.
  T modes enter first. Then, for $l > 1$, A modes start to enter...
  We get n modes for T with only one A mode. It is as if T is defined on a space lattice with n points, while A is defined on a single point.
- We expand in sine modes, since periodic boundary conditions would glue together the strong and weak coupling regions.
  Still not optimal...

Linear dilaton in d dimensions

Energy-momentum tensor

$T = -\partial bc - 2b\partial c - \frac{1}{\alpha'}\partial X^\mu \partial X_\mu + V^\mu \partial^2 X_\mu$ implies:

- Modified energy-momentum conservation: $\delta^d\left(\sum_n k_n^\mu + iV^\mu\right)$
- Modified conformal weights: $h(e^{ik\cdot X}) = \alpha'(k^2 + ik\cdot V)$
- Modified Q, since $J_B(z) = cT^m(z) + bc\partial c(z) + \frac{3}{2}\partial^2 c(z)$

For $d = 1$: $V = -\sqrt{\frac{25}{6\alpha'}}$.

The action

The action is: $S \equiv S_2 + S_3 = -\int \left(\frac{1}{2\alpha'} \Psi \star Q\Psi + \frac{g_o}{3} \Psi \star \Psi \star \Psi \right)$

For $l < 1$: $\Psi_0 = \int dp \left(T(p) e^{ipX} + i\mathcal{T}(p) e^{ipX} c_0 \right) c_1 |0\rangle$.

$\mathcal{T}$ drops from all expressions at this level.

Plugging in the action and evaluating using the rules leads to:

$$S_2 = -\frac{1}{2} \int dp_1\, dp_2\, 2\pi\delta(p_1 + p_2 + iV)\, T(p_1)\, T(p_2) \left(\left(p_1 + \frac{iV}{2} \right)^2 + m_0^2 \right)$$

$$S_3 = -\frac{g_o}{3} \int d^3p\, 2\pi\delta\left(\sum_{r=1}^{3} p_r + iV \right) K^{3-\alpha'(\sum_{r=1}^{3} p_r^2 + V^2)} \prod_{r=1}^{3} T(p_r)$$

$K = \frac{3\sqrt{3}}{4}$ $\qquad m_0^2 = \frac{V^2}{4} - \frac{1}{\alpha'} = \frac{1}{24\alpha'}$

As always, ***S is non-local***. Factors of iV due to the linear dilaton.

Lattice action

Redefining $T(p)$

- It could be nice to remove iV factors, making S_2 canonical.
- The reality condition should be imposed.

Both goals are achieved by redefining $p \to p - \frac{iV}{2}$.

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Mode expansion

- Expand: $T = \sqrt{\frac{2}{L}} \sum_{n=1}^{\infty} T_n \sin\left(\frac{\pi n x}{L}\right)$
- The action is: $S_2 = -\frac{1}{2} \sum_{n=1}^{N} \left(\frac{1}{24\alpha'} + \left(\frac{\pi n}{L}\right)^2\right) T_n^2$

$$S_3 = -\frac{g_o K^{3\left(1-\frac{\alpha' V^2}{4}\right)}}{3} \sum_{n_r=1}^{N} K^{-\alpha'\left(\frac{\pi}{L}\right)^2 (n_1^2+n_2^2+n_3^2)} f_{n_1,n_2,n_2} \prod_{r=1}^{3} T_{n_r}$$

Higher l_0 levels

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Gauge symmetry – four possible schemes

1. Include only $gh = 1$ modes in Siegel gauge.
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The first two schemes are probably too naive.
The next scheme leads to expressions that do not make sense regarding the fermionic integration, e.g., at $l_0 = 1$, we have an integration $d\mathcal{T}$, but no $\mathcal{T}$ in the action.

Strategy – Check experimentally the first three schemes

The first two are easy to evaluate, as they have no fermions.
On the other hand they are too naive.

The most promising scheme is ***scheme four***:
It uses the Siegel gauge, which makes level truncation senseful, since the level and energy are identified.
Also, scheme four ***properly treats the gauge structure***.

Scheme three include "fermions" – integrate them out.

Analytically continue $X_n \to e^{\frac{i\pi}{6}} X_n$, for all X, i,e., $X = T, A, \ldots$.

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Implementation

The analytical continuation makes S complex – it is no longer adequate as a measure function. Instead use ***phase quenching***:
Redefine $e^S \to |e^S|$ as the measure function and multiply by $\arg(e^S)$ inside the expectation values: $\langle \mathcal{O} \rangle = \frac{\int \mathcal{O} |e^S| e^{i\theta}}{\int |e^S| e^{i\theta}} = \frac{\langle \mathcal{O} e^{i\theta} \rangle_{\mathrm{PQ}}}{\langle e^{i\theta} \rangle_{\mathrm{PQ}}}$.
Problem: The method introduces an error.

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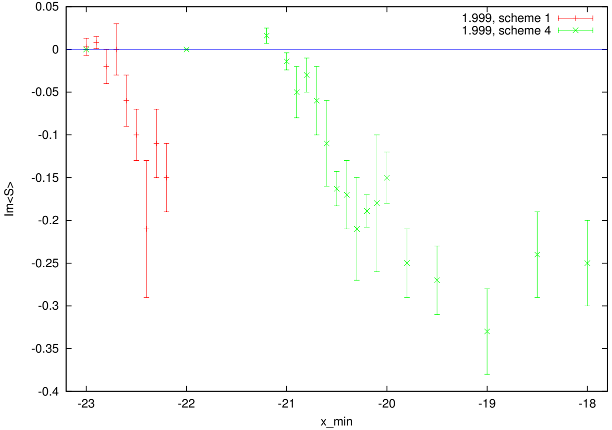
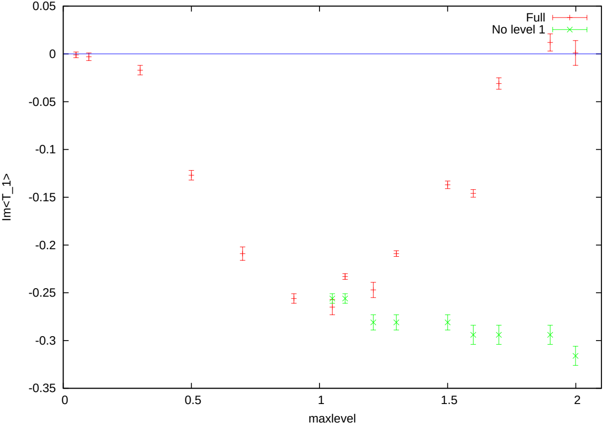
Fortran code generates phase-quenched configurations using a Metropolis algorithm.
Measured observables are the action S and the fields T_n.
Errors are estimated with the jackknife method.

Comparing scheme 1 and 4



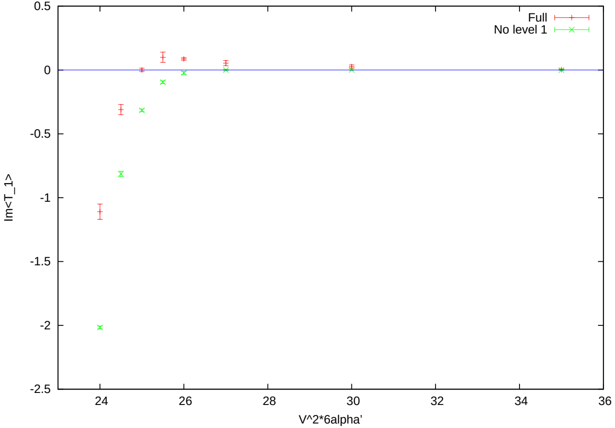
Seems that scheme four is indeed better than scheme one.
But is it good?

Dependence of $\Im(T_1)$ on the level



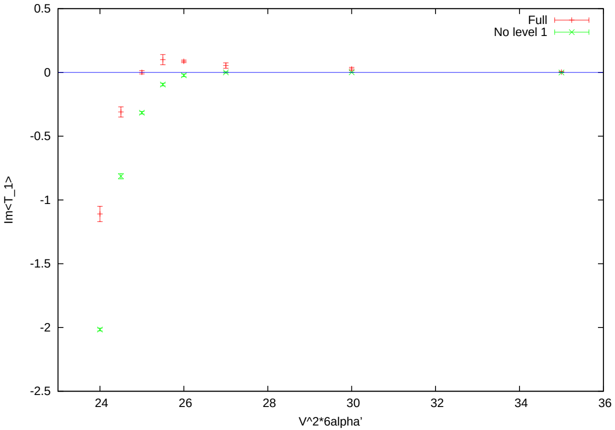
Red: $\Im(T_1)$ *as a function of* l. *For* $l > 1$ *the* $l_0 = 1$ *modes enter*
Green: *The same, not including the* $l_0 = 1$ *modes*

Dependence of $\Im(T_1)$ on the dilaton slope



Red: $\Im(T_1)$ *as a function of* $6\alpha' V^2$ *for* $l = 2$.
Green: *The same, not including the* $l_0 = 1$ *modes.*

Dependence of $\Im(T_1)$ on the dilaton slope



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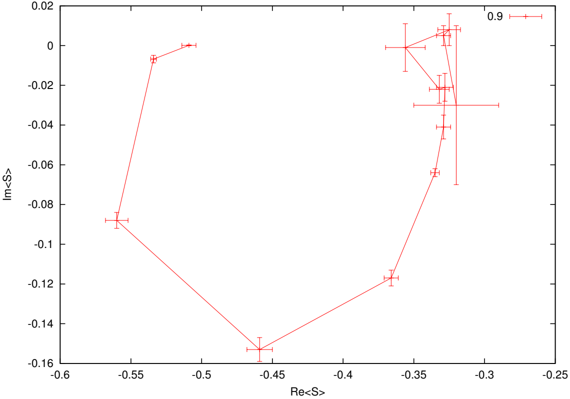
Unfortunately, these results ***change when we change*** x_{min} ***and*** x_{max}.

Taking $x_{max} \to \infty$



The action in complex plane for $L = 6$ and a single mode.
It begins at low x_{min} as a free theory: $S = -\frac{1}{2}$.
Travels in the complex plane and ends up at $S \approx -\frac{1}{3}$.

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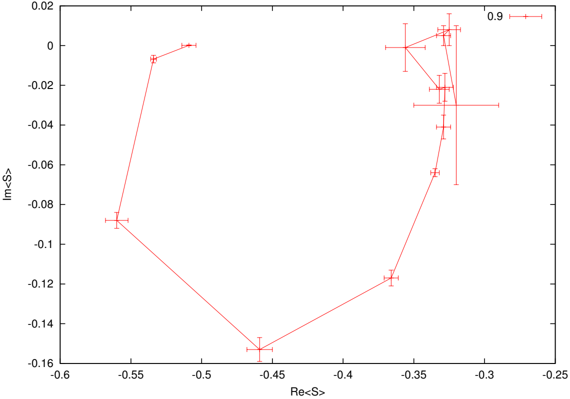

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Encouraging result. Can be obtain analytically.
What happens when more modes are added? Dilaton slope dependence?

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Future directions

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- Study convergence upon taking $-x_{min}, x_{max} \to \infty$.
- Implement in the continuous basis.
- Repeat at $d = 2$ and compare to known results.

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Is the approach sensible?

- What about the quantum BV master equation?
- Renormalizability?
- Algorithms for (the non-local theory) at higher level?