String Field Theory on Intersecting D-branes

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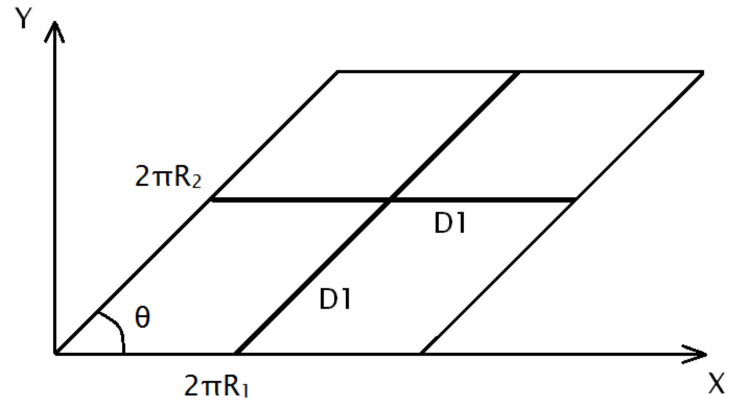
- Untwisted sectors 11 and 22
- Twisted sectors 12 and 21
- Action and 3-vertex
- Conservation Laws
- Conclusion

- Lump solutions on torus

Torus and D-brane geometry

We consider bosonic string theory with 2 compact dimensions

$$X^i,\ i = 0, \ldots, 23,$$
$$X^{24} \equiv X,$$
$$X^{25} \equiv Y.$$



We complexify the coordinates

$$Z = X + iY$$
$$\bar{Z} = X - iY$$

The two compact dimesions form a torus with radii R_1 and R_2 and angle $\theta = \pi\alpha$

The periodicities of the coordinates are

$$(X,Y) \sim (X + 2\pi R_1, Y),\ (X,Y) \sim (X + 2\pi R_2 \cos\theta, Y - 2\pi R_2 \sin\theta)$$
$$Z \sim Z + 2\pi R_1,\ Z \sim Z + 2\pi R_2 e^{-i\pi\alpha}.$$

The boundary conditions for string endind on the first D-brane are

$$Y = 0,$$
$$\partial_\sigma X = 0$$

and on the second D-brane

$$\tan\theta X + Y = 0,$$
$$\partial_\sigma (X - \tan\theta Y) = 0.$$

In the complex coordinates

$$\partial_\sigma Re[Z] = 0,$$
$$Im[Z] = 0,$$
$$\partial_\sigma Re[e^{i\pi\alpha} Z] = 0,$$
$$Im[e^{i\pi\alpha} Z] = 0.$$

The string theory has four different sectors denoted 11, 12, 21and 22.

Untwisted sectors 11 and 22

Mode expansions in sector 11

$$Z_{11}(z,\bar{z}) = z_0 - ip\ln|z|^2 + w\tilde{R}_2 \ln\frac{z}{\bar{z}} + i\sum_{k\neq 0}\frac{1}{k}\left(\frac{x_k}{z^k} + \frac{\bar{x}_k}{\bar{z}^k}\right),$$

$$\bar{Z}_{11}(z,\bar{z}) = \bar{z}_0 - ip\ln|z|^2 - w\tilde{R}_2 \ln\frac{z}{\bar{z}} + i\sum_{k\neq 0}\frac{1}{k}\left(\frac{\bar{x}_k}{z^k} + \frac{x_k}{\bar{z}^k}\right),$$

where the oscillators are

$$x_k = \frac{1}{\sqrt{2}}(\alpha_k^x + i\alpha_k^y),$$

$$\bar{x}_k = \frac{1}{\sqrt{2}}(\alpha_k^x - i\alpha_k^y)$$

and have commutation relations

$$[x_m, \bar{x}_n] = m\delta_{m+n,0}$$

The zero modes describe combination of momenta and windind number

$$x_0 = p_L = \bar{p}_R = p + iR_2 \sin\theta w,$$
$$\bar{x}_0 = \bar{p}_L = p_R = p - iR_2 \sin\theta w.$$

Vertex operators corresponding to states with nonzero momentum and winding are written as

$$|k_L, k_R\rangle \sim : e^{\frac{i}{2}(\bar{k}_L Z_L + k_L \bar{Z}_R + \bar{k}_R Z_R + k_R \bar{Z}_L)} : .$$

The correlation functions are

$$\left\langle \prod_i : e^{\frac{i}{2}(\bar{k}_{iL} Z_L + k_{iL} \bar{Z}_R + \bar{k}_{iR} Z_R + k_{iR} \bar{Z}_L)}(z_i) : \right\rangle = \prod_{i<j} |z_i - z_j|^{k_{iL}\bar{k}_{jL} + \bar{k}_{iL} k_{jL}} 2\pi R_1 \delta_{\sum_i k_{iL}} \delta_{\sum_i \bar{k}_{iL}} .$$

where

$$k_{iL}\bar{k}_{jL} + \bar{k}_{iL} k_{jL} = 2(k_i k_j + w_i w_j R^2 \sin^2\theta)$$

The fields in sector 22 are obtained by rotation $Z_{22} = e^{-i\pi\alpha} Z_{11}$

$$Z_{22}(z,\bar{z}) = e^{-i\pi\alpha}\left(\hat{z}_0 - i\hat{p}\ln|z|^2 + \hat{w}\tilde{R}_1 \ln\frac{z}{\bar{z}} + i\sum_{k\in Z}\frac{1}{k}\left(\frac{\hat{x}_k}{z^k} + \frac{\hat{\bar{x}}_k}{\bar{z}^k}\right)\right),$$

$$\bar{Z}_{22}(z,\bar{z}) = e^{i\pi\alpha}\left(\hat{\bar{z}}_0 - i\hat{p}\ln|z|^2 - \hat{w}\tilde{R}_1 \ln\frac{z}{\bar{z}} + i\sum_{k\in Z}\frac{1}{k}\left(\frac{\hat{\bar{x}}_k}{z^k} + \frac{\hat{x}_k}{\bar{z}^k}\right)\right).$$

In both sectors we split the fields into holomorphic and antiholomorphic parts and make identification

$$Z_L(z') = \bar{Z}_R(\bar{z}),\ \bar{Z}_L(z') = Z_R(\bar{z})\ (Im[z'] < 0).$$

Then the primary field $\partial Z(z)$ is holomorphic in the entire complex plane in sector 11, but it has branch cut along the real axis in sector 22.

Twisted sectors 12 and 21

The mode expansion in sector 12 is

$$Z(z,\bar{z}) = i\sum_{k\in Z}\left(\frac{x_{k+\alpha}}{k+\alpha}\frac{1}{z^{k+\alpha}} + \frac{\bar{x}_{k-\alpha}}{k-\alpha}\frac{1}{\bar{z}^{k-\alpha}}\right),$$

$$\bar{Z}(z,\bar{z}) = i\sum_{k\in Z}\left(\frac{\bar{x}_{k-\alpha}}{k-\alpha}\frac{1}{z^{k-\alpha}} + \frac{x_{k+\alpha}}{k+\alpha}\frac{1}{\bar{z}^{k+\alpha}}\right).$$

Oscillators $x_{k+\alpha}$ and $\bar{x}_{k-\alpha}$ are labeled by integers shifted by the angle α Their commutation relation is

$$[\bar{x}_{m-\alpha}, x_{n+\alpha}] = (m-\alpha)\delta_{m+n,0}.$$

There is no momentum or winding number in this sector.

After making the doubling trick the field $\partial Z(z)$ has branch cut along the the negative part of real axis.

The fields in sector 21 are

$$Z(z,\bar{z}) = ie^{-i\pi\alpha}\sum_{k\in Z}\left(\frac{\hat{x}_{k-\alpha}}{k-\alpha}\frac{1}{z^{k-\alpha}} + \frac{\hat{\bar{x}}_{k+\alpha}}{k+\alpha}\frac{1}{\bar{z}^{k+\alpha}}\right),$$

$$\bar{Z}(z,\bar{z}) = ie^{i\pi\alpha}\sum_{k\in Z}\left(\frac{\hat{\bar{x}}_{k+\alpha}}{k+\alpha}\frac{1}{z^{k+\alpha}} + \frac{\hat{x}_{k-\alpha}}{k-\alpha}\frac{1}{\bar{z}^{k-\alpha}}\right).$$

This time after making the doubling trick the field $\partial Z(z)$ has branch cut along the the positive part of real axis.

Up to a phase factor the fields are very similar, so from now on we shall present results only for sector 12.

The lowest state is not given by the SL(2,R) invariant vacuum $|0\rangle$ but by a twisted vacuum

$$\sigma_\alpha(0)|0\rangle \equiv |\sigma_\alpha\rangle$$

The conformal weight of the twist operator is

$$\frac{1}{2}(\alpha - \alpha^2) = h_\alpha$$

The lightest states and their masses are

$$c_1|\sigma_\alpha\rangle, \quad m^2 = -1 + \frac{1}{2}(\alpha - \alpha^2)$$

$$(\bar{x}_{-\alpha})^n c_1|\sigma_\alpha\rangle, \quad m^2 = -1 + \frac{1}{2}(\alpha - \alpha^2) + n\alpha$$

$$(x_{-1+\alpha})^n c_1|\sigma_\alpha\rangle, \quad m^2 = -1 + \frac{1}{2}(\alpha - \alpha^2) + n(1-\alpha)$$

so there are always at least three states with negative mass squared.

The basic correlation function is

$$\langle Z(z_1,\bar{z_1})\bar{Z}(z_2,\bar{z}_2)\rangle = \langle \sigma_{-\alpha}|R(Z(z_1,\bar{z_1})\bar{Z}(z_2,\bar{z}_2))|\sigma_\alpha\rangle$$

It can be evaluated using the mode expansion

$$\langle Z(z_1,\bar{z_1})\bar{Z}(z_2,\bar{z}_2)\rangle = \frac{1}{\alpha}\left(\frac{z_2}{z_1-z_2}\right)^{\alpha} F\left(\alpha,\alpha,1+\alpha,-\frac{z_2}{z_1-z_2}\right)$$
$$+\frac{1}{1-\alpha}\left(\frac{\bar{z}_2}{\bar{z}_1-\bar{z}_2}\right)^{1-\alpha} F\left(1-\alpha,1-\alpha,2-\alpha,-\frac{\bar{z}_2}{\bar{z}_1-\bar{z}_2}\right).$$

F represents hypergeometric function.

The general correlation function with twist operators placed at arbitrary points is fully determined by general properties of 4-point functions, which depend only on the invariant ratio

$$z_C = \frac{(z_1-x_1)(z_2-x_2)}{(z_1-z_2)(x_1-x_2)}$$

$$\langle Z(z_1,\bar{z_1})\bar{Z}(z_2,\bar{z}_2)\rangle_{(x_1,x_2)} = \langle Z(z_1,\bar{z_1})\bar{Z}(z_2\bar{z}_2)\sigma_\alpha(x_1)\sigma_{-\alpha}(x_2)\rangle =$$
$$= \frac{1}{\alpha}(x_1-x_2)^{-2h_\alpha}(z_C)^\alpha F\left(\alpha,\alpha,1+\alpha,-z_C\right)$$
$$+\frac{1}{1-\alpha}(x_1-x_2)^{-2h_\alpha}(\bar{z}_C)^{1-\alpha}F\left(1-\alpha,1-\alpha,2-\alpha,-\bar{z}_C\right),$$

The correlator for the holomorphic parts of the fields is

$$\langle Z_L(z_1)\bar{Z}_L(z_2)\rangle \;\; = \;\; \frac{1}{\alpha}(x_1-x_2)^{-2h_\alpha}(z_C)^\alpha F\left(\alpha,\alpha,1+\alpha,-z_C\right)$$

Action and 3-vertex

The string field has matrix structure

$$\Psi = \begin{pmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{pmatrix}.$$

SFT action is given by

$$S = -\frac{1}{2}\int Tr(\Psi Q_B \Psi) - \frac{1}{3}\int Tr(\Psi * \Psi * \Psi).$$

The kinetic and interaction terms separates into parts

$$\frac{1}{2}\int Tr(\Psi Q_B \Psi) = \frac{1}{2}\int (\Psi_{11} Q_B \Psi_{11} + 2\Psi_{21} Q_B \Psi_{12} + \Psi_{22} Q_B \Psi_{22})$$

$$\frac{1}{3}\int Tr(\Psi * \Psi * \Psi) = \frac{1}{3}\int (\Psi_{11}^3 + 3\Psi_{11} * \Psi_{12} * \Psi_{21} + 3\Psi_{22} * \Psi_{21} * \Psi_{12} + \Psi_{22}^3).$$

The main object of interest is the 3-vertex which included twisted fields.

The simplest fields are

$$\Psi_{11} = \sum_{n,w} t_{11}(n,w) c_1 |n,w\rangle,$$

$$\Psi_{12} = t_{12} c_1 |\sigma_\alpha\rangle,$$

$$\Psi_{21} = t_{12}^* c_1 |\sigma_{-\alpha}\rangle.$$

The definition of 3-vertex is

$$\int \Psi_1 * \Psi_2 * \Psi_3 = \langle f_1 \circ V_1(0) f_2 \circ V_2(0) f_3 \circ V_3(0) \rangle,$$

where

$$f_1(z) = \tan\left(\frac{2}{3}\arctan z + \frac{\pi}{3}\right),$$

$$f_2(z) = \tan\left(\frac{2}{3}\arctan z\right),$$

$$f_3(z) = \tan\left(\frac{2}{3}\arctan z - \frac{\pi}{3}\right).$$

The vertex with our fields is

$$\int \Psi_{11} * \Psi_{12} * \Psi_{21} = \sum_{n,w} t_{11}(n,w) |t_{12}|^2 \left\langle f_1 \circ e^{ik.Z}(0), f_2 \circ \sigma_\alpha(0), f_3 \circ \sigma_{-\alpha}(0) \right\rangle \langle ghost \rangle$$

so we need correlation fuction

$$\langle e^{ik.Z}(z) \sigma_\alpha(x_1) \sigma_{-\alpha}(x_2) \rangle$$

Because the Polyakov action is gaussian, it can be calculated using generating functional

$$Z[J] = \left\langle e^{i \int d^2\sigma J(\sigma).X(\sigma)} \right\rangle_{(x_1,x_2)} = N e^{-\frac{1}{2} \int d^2\sigma d^2\sigma' J(\sigma).G(\sigma,\sigma').J(\sigma'))}.$$

The Green's functions are proportional to the two-point functions, for example

$$G_{L\bar{L}}(z_1, z_2) = \frac{1}{1-\alpha} (z_C)^{1-\alpha} F\left(1-\alpha, 1-\alpha, 2-\alpha, -z_C\right)$$

The function is divergent as $z_1 \to z_2$, so it has to regularized before being evaluated at the same points.

$$G^r_{L\bar{L}}(z,z) = \ln\frac{(z-x_1)(z-x_2)}{(x_1-x_2)} - \gamma_E - \psi(1-\alpha).$$

γ_E is Euler's number and ψ is digamma function.
The final expression for the correlator is

$$\langle e^{ik.Z}(z)\sigma_\alpha(x_1)\sigma_{-\alpha}(x_2)\rangle =$$

$$= (x_1-x_2)^{-2h_\alpha+k_L\bar{k}_L}(z-x_1)^{-k_L\bar{k}_L}(z-x_2)^{-k_L\bar{k}_L}e^{k_L\bar{k}_L\left(\gamma_E+\frac{1}{2}(\psi(\alpha)+\psi(1-\alpha))\right)}$$

and the 3-vertex is equal to

$$\int \Psi_{11} * \Psi_{12} * \Psi_{21} =$$

$$= \sum_{n,w} t_{11}(n,w)|t_{12}|^2 K^{3-2h_\alpha-\left(\frac{n^2}{R_1^2}+w^2\tilde{R_2}^2\right)} e^{\left(\frac{n^2}{R_1^2}+w^2\tilde{R_2}^2\right)\left(\gamma_E+\frac{1}{2}(\psi(\alpha)+\psi(1-\alpha))\right)}$$

Conservation Laws

To compute interaction term for fields at higher levels we need conservation laws in twisted sectors.

For weight 1 current $j(z) = i\partial Z(z)$ and scalar function $g(z)$ regular at infinty we have

$$\langle V_3| \oint dz g(z) j(z) = 0.$$

Deforming the contour we get

$$\langle V_3| \left(\sum_{i=1}^{3} \oint dz_i j(z_i) g^{(i)}(z_i) \right) = 0$$

where $g^{(i)}(z) = g(f_i(z))$

The scalar function has to be chosen in such a way that it eliminates the branch cuts of $\partial Z(z)$.

For example we derive conservation law for $x_{-1+\alpha}$ at 11-12-21 vertex. The branch cut is localized from $-\sqrt{3}$ to 0 and the correct scalar function is

$$g_1(z) = 2^{1-\alpha}\sqrt{3}^{3\alpha-3} z^{-1+\alpha}(z+\sqrt{3})^{1-\alpha}$$

This function has the branch cut at the same place, but with the opposite phase change.

The expansions of $g_1^{(i)}$ are

$$g_1^{(1)}(z_1) = K^{-1+\alpha} - K^{-2+\alpha}(1-\alpha)z_1 + \frac{1}{2}K^{-3+\alpha}(1-\alpha)^2 z_1^2 + O(z_1^3),$$

$$g^{(2)}(z_2) = z_2^{-1+\alpha} + \frac{1-\alpha}{2}K^{-1}z_2^{\alpha} + \frac{1}{16}K^{-2}(5-7\alpha+2\alpha^2)z_2^{1+\alpha} + O(z_2^{2+\alpha}),$$

$$g^{(3)}(z_3) = (-1)^{\alpha}\left(-K^{-2+2\alpha}z_3^{1-\alpha} + \frac{-1+\alpha}{2}K^{-3+2\alpha}z_3^{2-\alpha}\right) + O(z_3^{3-\alpha}).$$

Integrating the expansions with correct powers of z leads to conservation law

$$0 = \langle V_3| \left(K^{-1+\alpha} x_0 - K^{-2+\alpha}(1-\alpha) x_1 + \frac{1}{2} K^{-3+\alpha}(1-\alpha)^2 x_2 \right)^{(1)}$$
$$+\langle V_3| \left(x_{-1+\alpha} + \frac{1-\alpha}{2} K^{-1} x_\alpha + \frac{1}{16} K^{-2}(5 - 7\alpha + 2\alpha^2) x_{1+\alpha} \right)^{(2)}$$
$$+\langle V_3| \left(-K^{-2+2\alpha} \hat{x}_{1-\alpha} + \frac{-1+\alpha}{2} K^{-3+2\alpha} \hat{x}_{2-\alpha} \right)^{(3)} .$$

There are 12 types of conservation laws in total, but they can be derived just from 3 types by replacing the oscillators and changing the angle.

Conclusions

- We are able to determine the correlation functions and compute the action for the system of intersecting D1-branes
- There are problems with solving the equations of motion (it is unclear how to fix symmetries)
- There are problems with interpretation of the solutions – the energy is not precise at low levels and there is lot of possible final states. Possible solution is to use Ellwood´s invariants.

- As a side project to undestand various problems on a simpler example we are looking for lump solutions on a torus.

Lump solutions on torus

We use standard bosonic OSFT on the same torus as before.

We consider states build by acting with oscillators

$$L'_{-1}, L'_{-2}, \dots, L^X_{-1}, L^X_{-2}, \dots, L^Y_{-1}, L^Y_{-2}, \dots, c_1, c_{-1}, \dots, b_{-2}, c_{-3}, \dots$$

on vacua with nontrivial momenta in the X and Y direction

$$e^{i(k_X X + k_Y Y)}(0)|0\rangle$$

and on even primary states made of $\alpha^{X,Y}_{-k}$.

To reduce the ambiguity of the solution given by the translation symmetry on the torus we impose the parity symmetry

$$(X, Y) \to (-X, -Y)$$

so the only surviving momentum states are

$$\frac{1}{2}\left(e^{i(k_X X + k_Y Y)}(0) + e^{-i(k_X X + k_Y Y)}(0)\right)|0\rangle = \cos\left(k_X X + k_Y Y\right)(0)|0\rangle$$

To identify the solution we can use energy, Ellwood´s invariants and we can plot the value of tachyon field.

The energy has to be correctly normalized.

The action of string field theory is

$$S = -V_2 \tau_2 \left(\tfrac{1}{2} \int \Psi Q_B \Psi + \tfrac{1}{3} \int \Psi * \Psi * \Psi \right)$$

which we have already divided by the volume of the noncompact dimensions.

We normalize the energy (or mass) of the lump to be

$$E = \tfrac{V_2 \tau_2}{\tau_0} (1 - 2\pi^2 S)$$

The tension τ_2 of the D2 brane is proportional to

$$\tau_2 = (2\pi)^2 \tau_0$$

So the energy is

$$E = R_1 R_2 \sin\theta (1 - 2\pi^2 S)$$

In this convention the energies of the following objects are

- Tachyon vacuum E=0
- D0-brane E=1
- D1-brane E=length/2π
- D2-brane E=area/$(2\pi)^2$

The Ellwood´s invariants are defined by product of the string field with closed string vertex operator (with ghost number 2 and zero conformal weights) and identity string field

$$W(\Psi, V) = \langle I|V(i)|\Psi\rangle$$

We take the vertex operator to be

$$V^{\mu\nu} = c\bar{c}(i)\partial X^\mu \bar{\partial} X^\nu (i)$$

We use such normalization that the V^{00} invariant converges to energy, the presented value is equal to

$$W = R_1 R_2 \sin\theta (1 + 4\pi i \langle I | V(i) | \Psi \rangle)$$

We found that the components of the invariant orthogonal to the torus are in good agreement with the energy, but the components along the torus are not corvergent in the level truncation scheme. However the likely cause is a mistake in our program, so we shall not present them.

We shall take as our example a torus with $R_1=R_2=2.4$ and $\theta=\pi/3$. The results are computed at level 4.

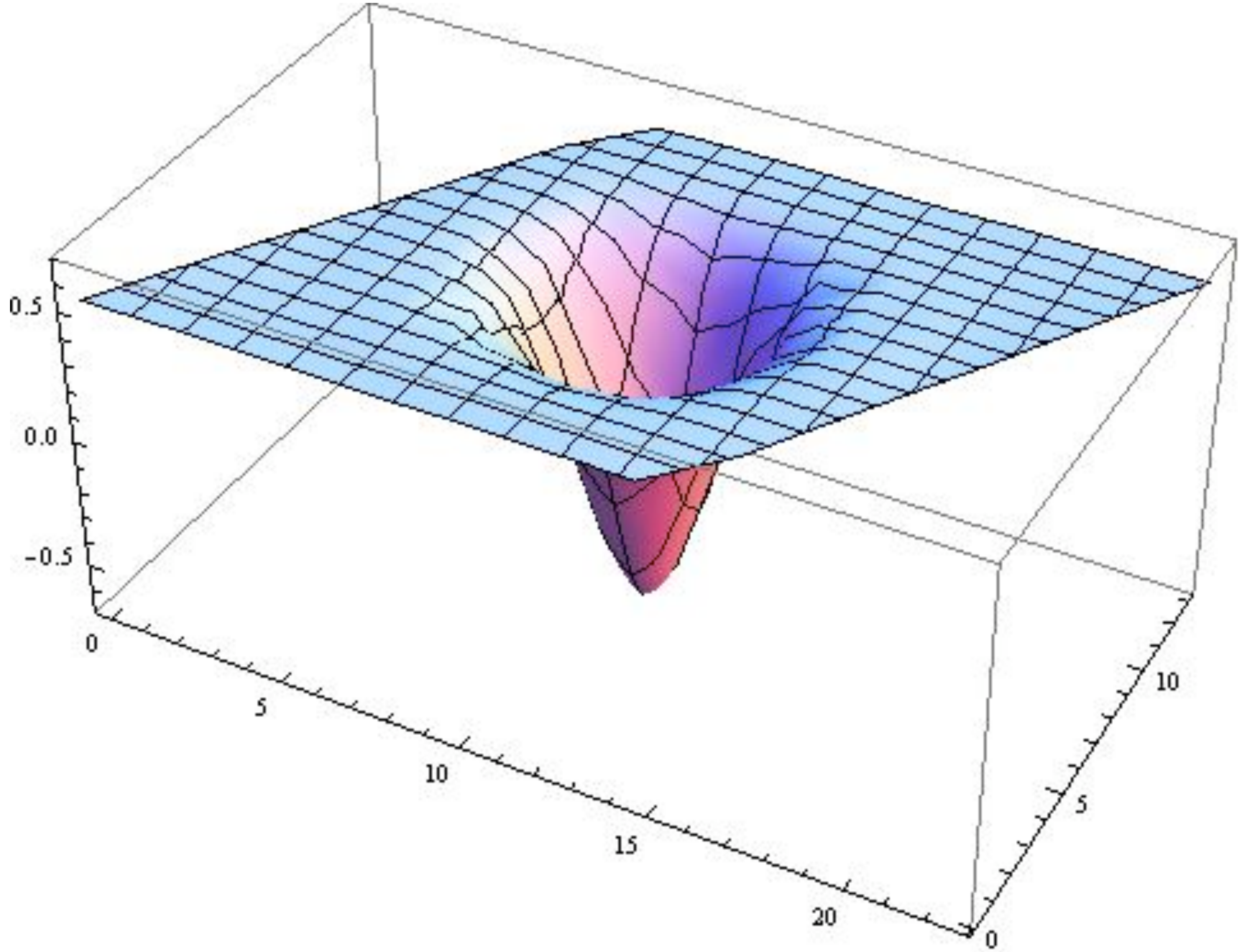
We can (in principle) find all configurations of D0 and D1 branes with energy lower than of the original D2 brane plus some unexpected solutions.

D0-brane

E=1.12

E_{exp}=1

W=1.27

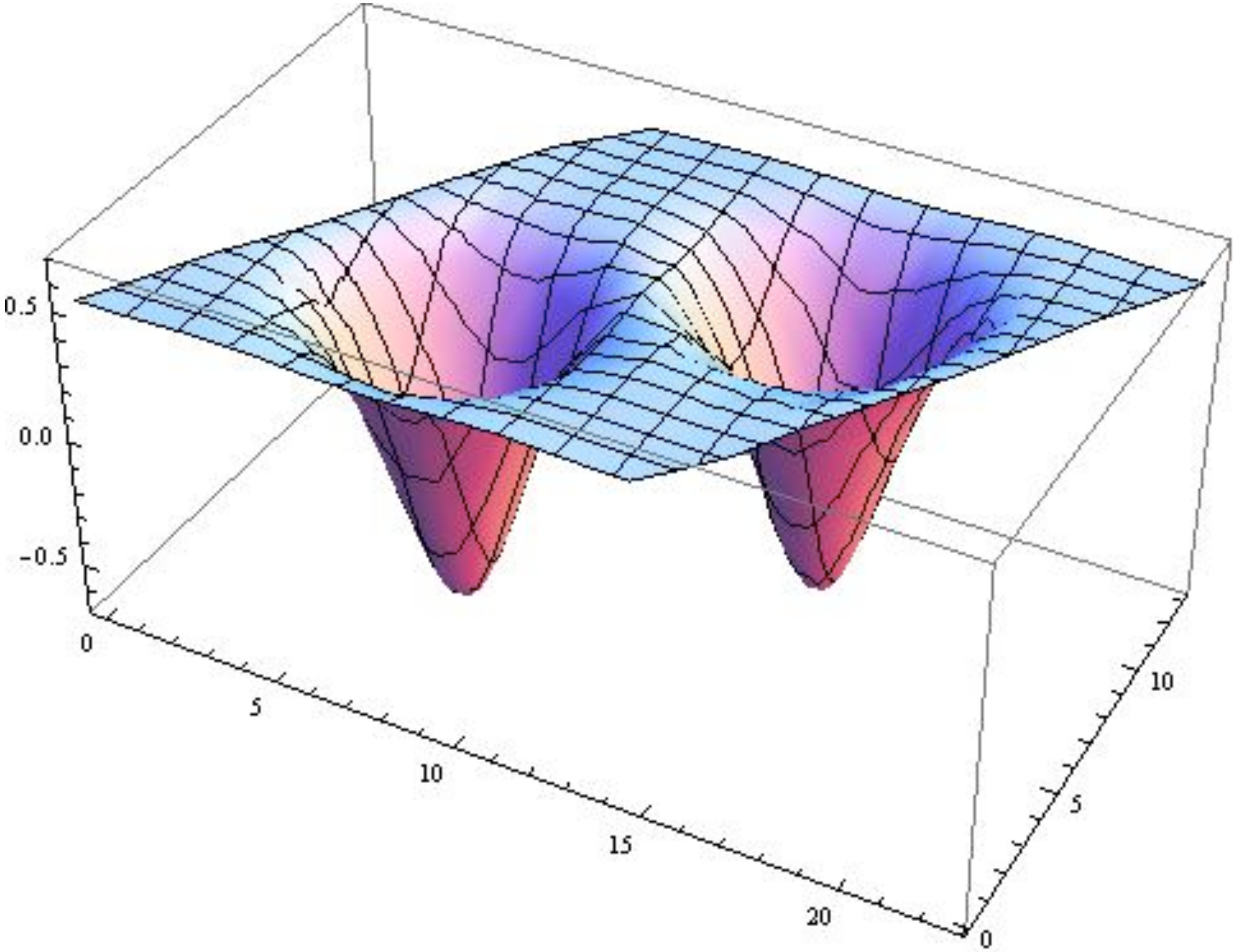


2 D0-branes

E=2.20

E_{exp}=2

W=2.21

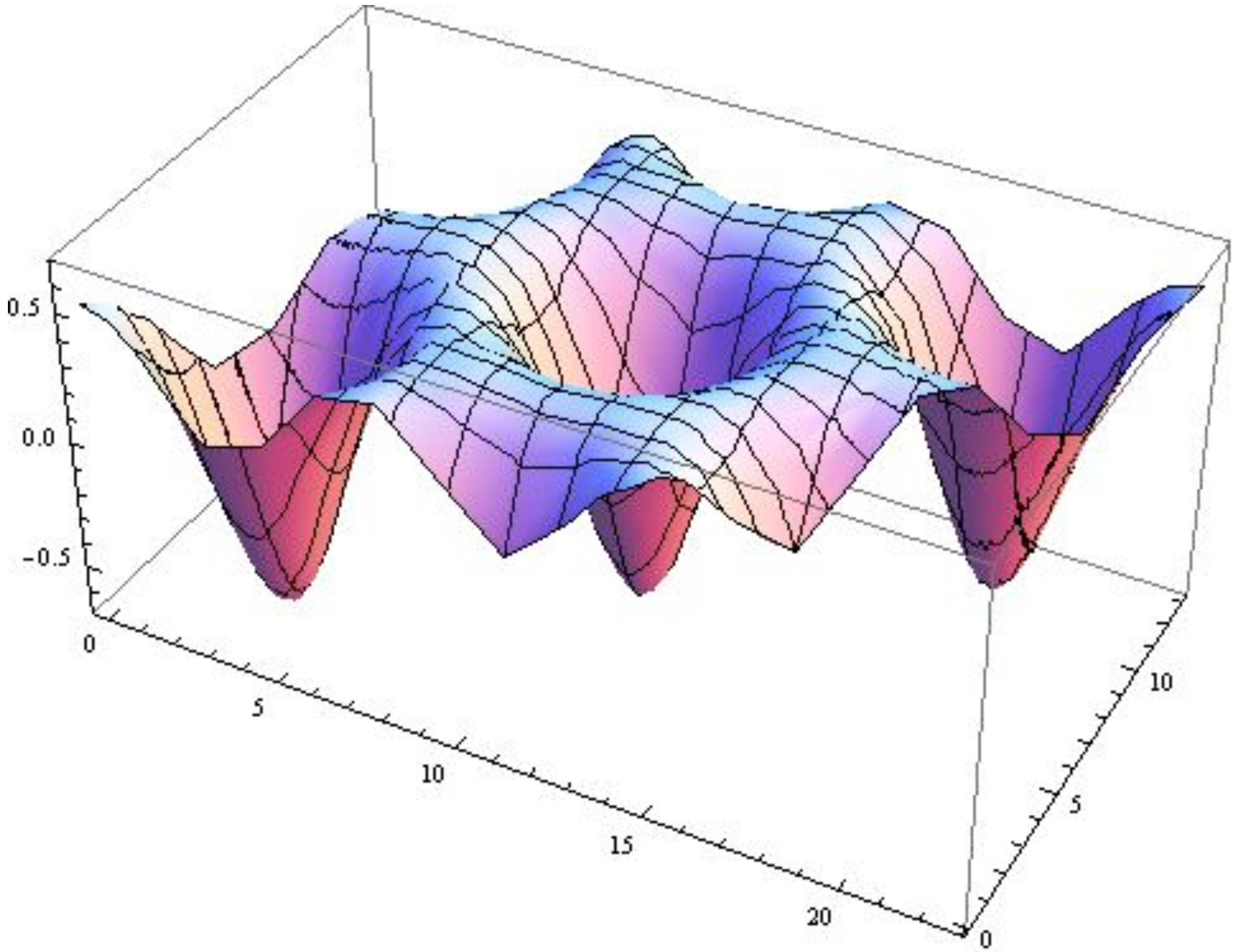


3 D0-branes

$E=3.26$

$E_{exp}=3$

$W=3.14$

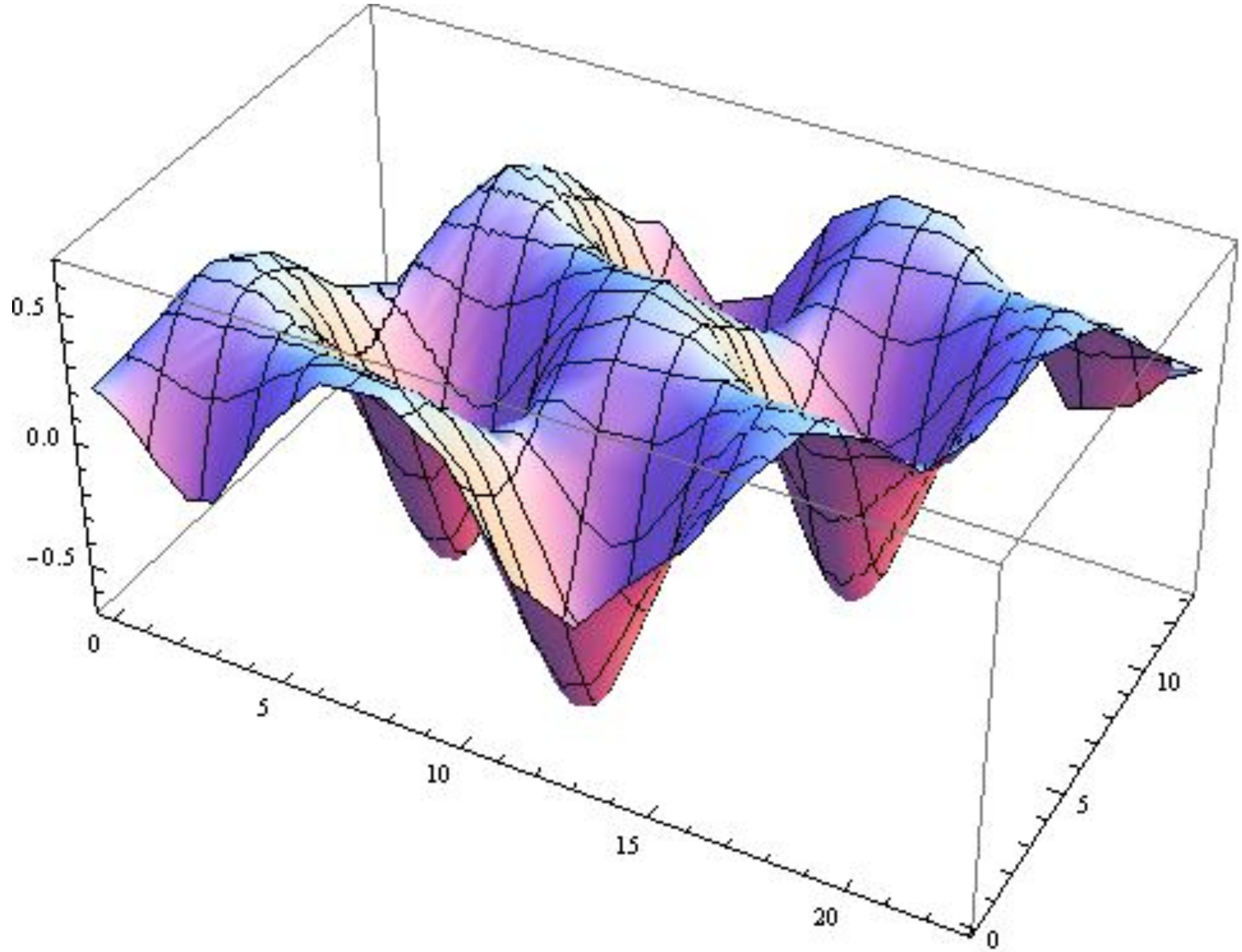


D0-brane

E=4.25

E_{exp}=4

W=4.08

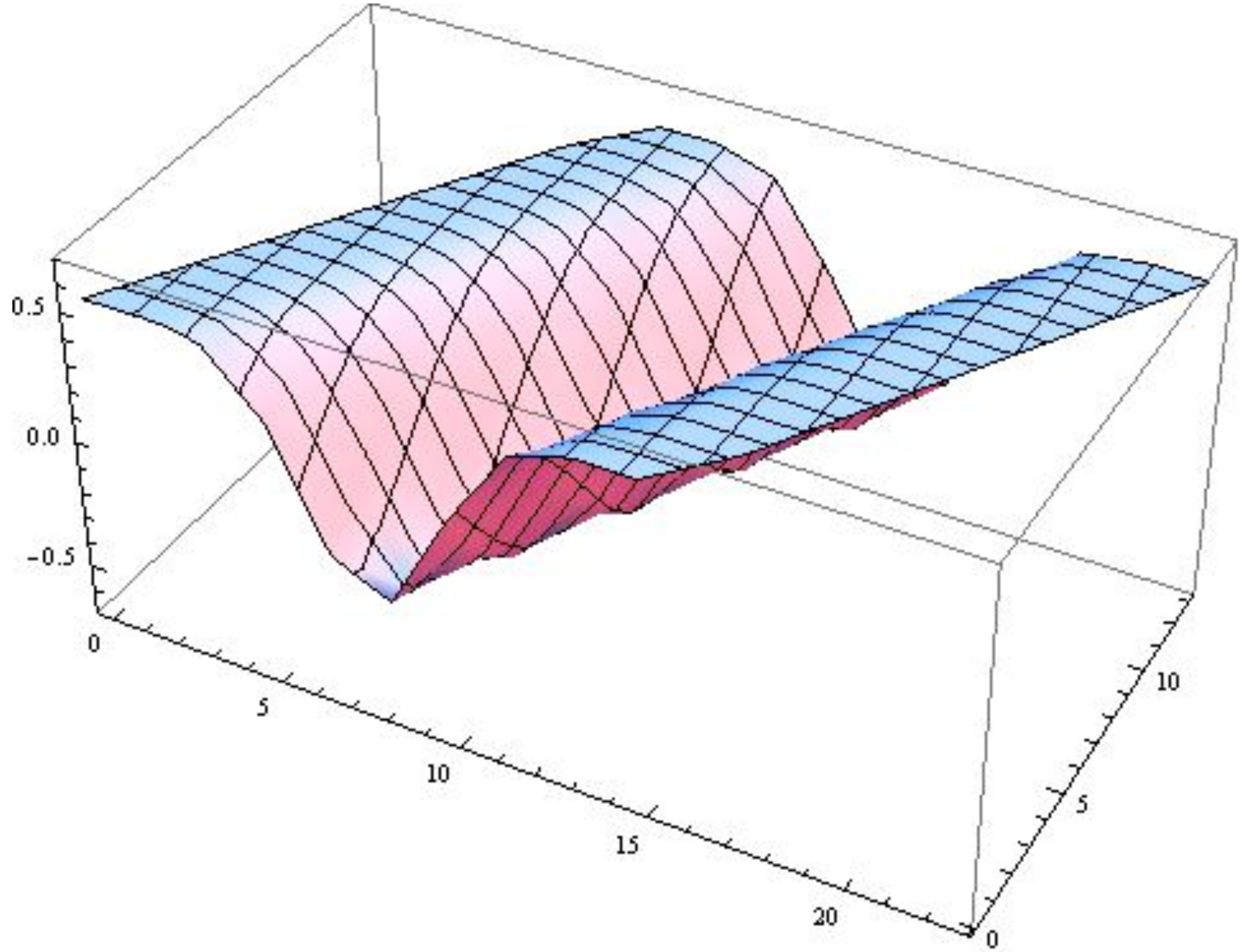


D1-brane, winding (0,1)

$E=2.53$

$E_{exp}=2.4$

$W=2.61$

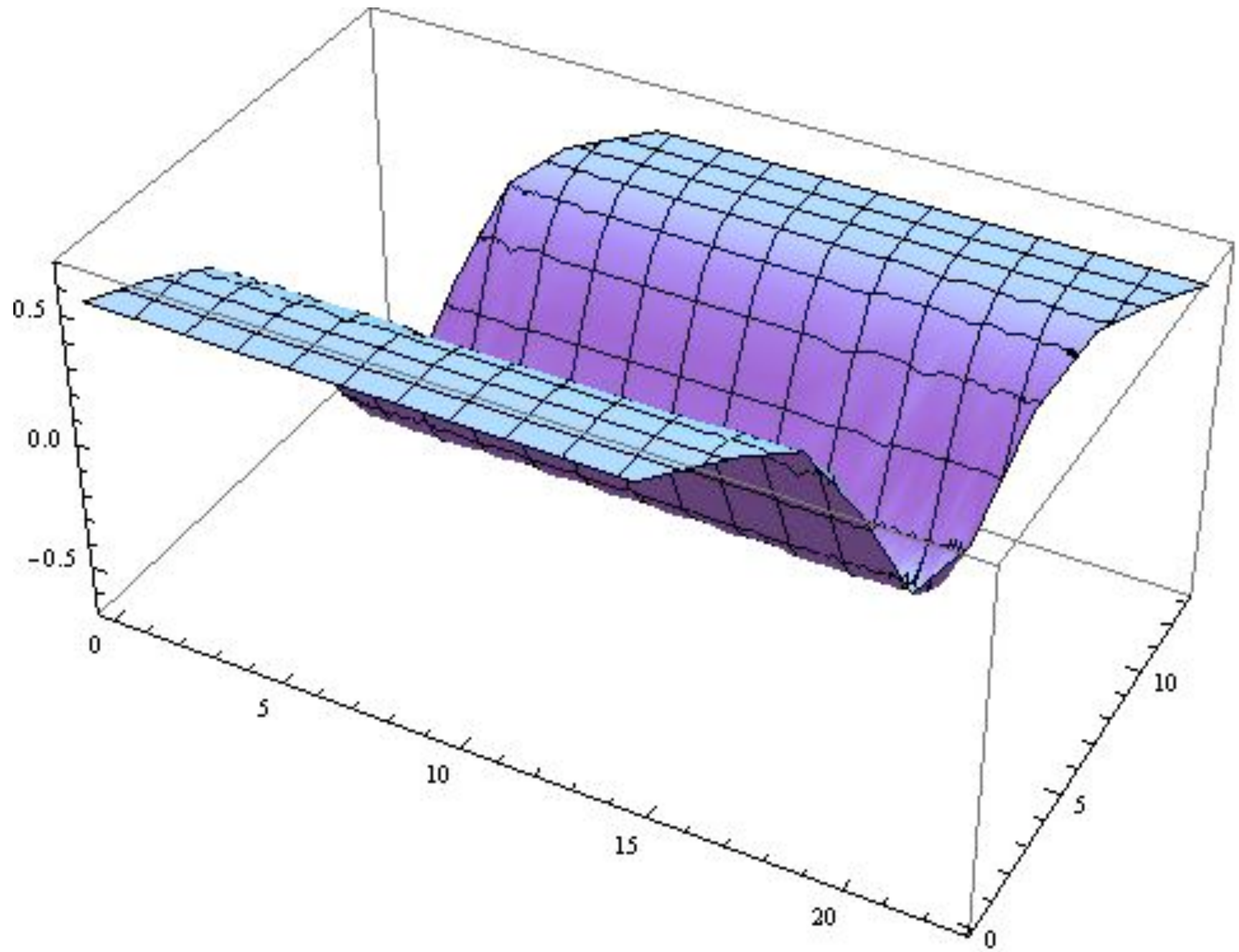


D1-brane, winding (1,0)

E=2.50

E_{exp}=2.4

W=2.57

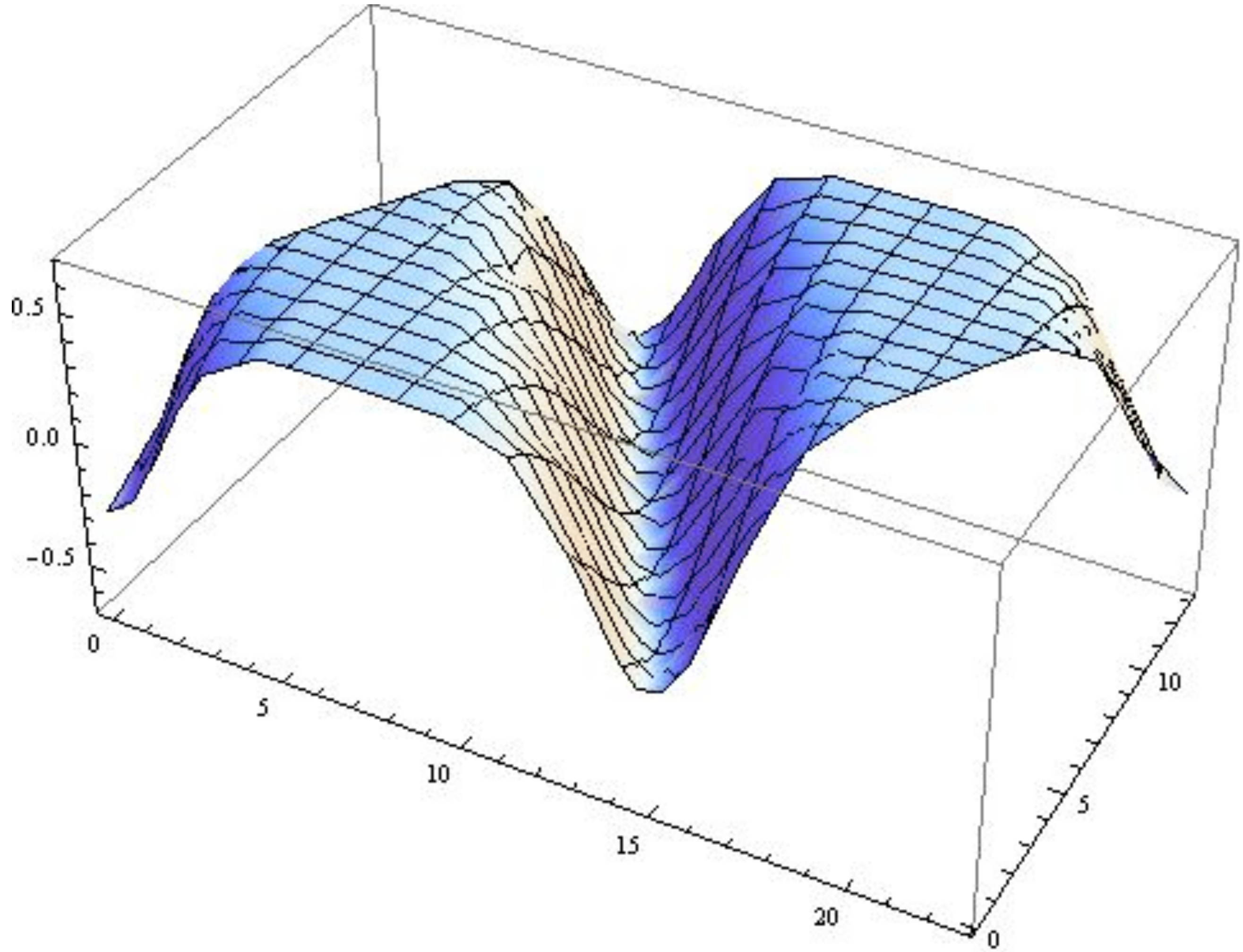


D1-brane, winding (1,-1)

E=2.53

E_{exp}=2.4

W=2.61

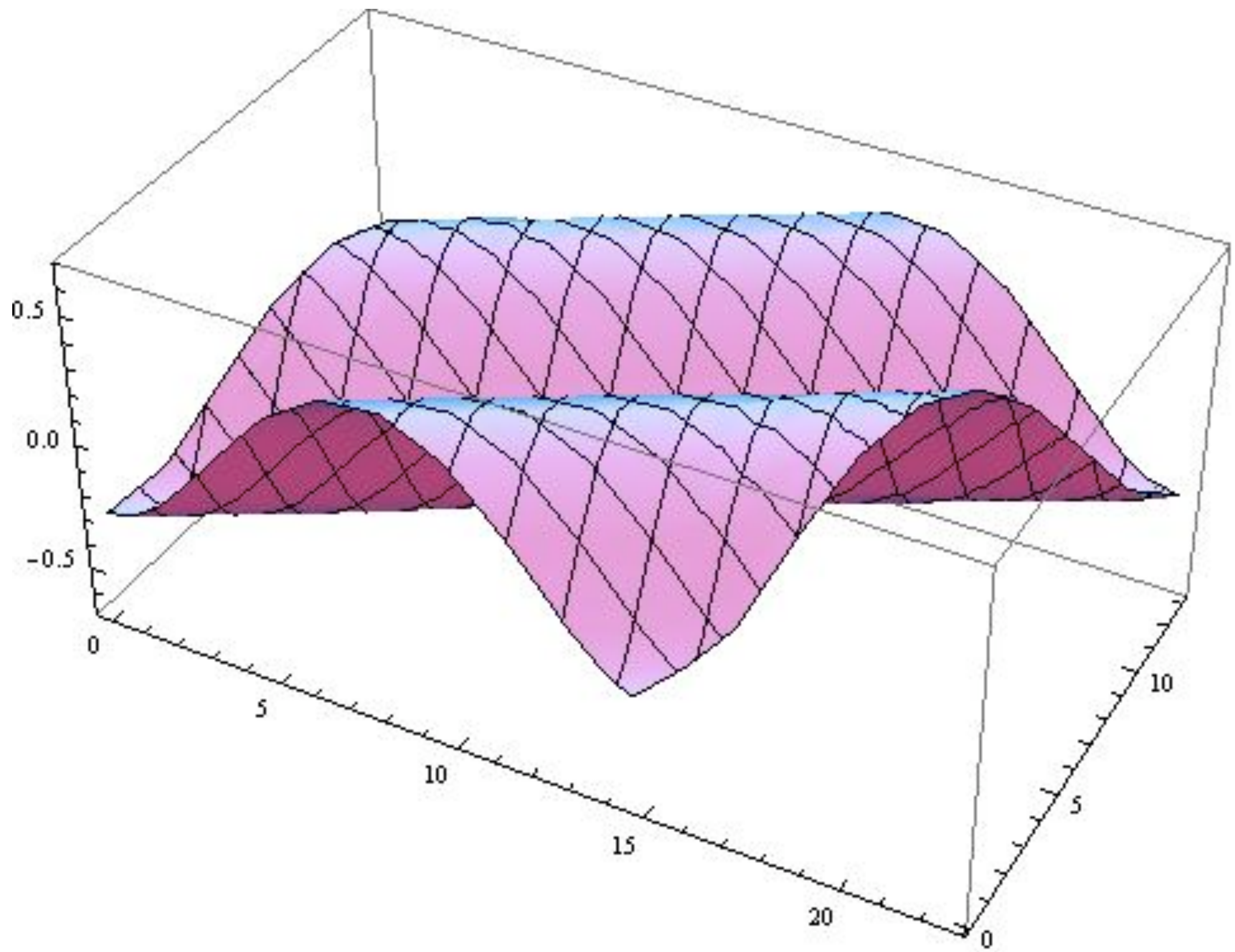


D1-brane, winding (1,1)

E=4.33

E_{exp}=4.16

W=4.28

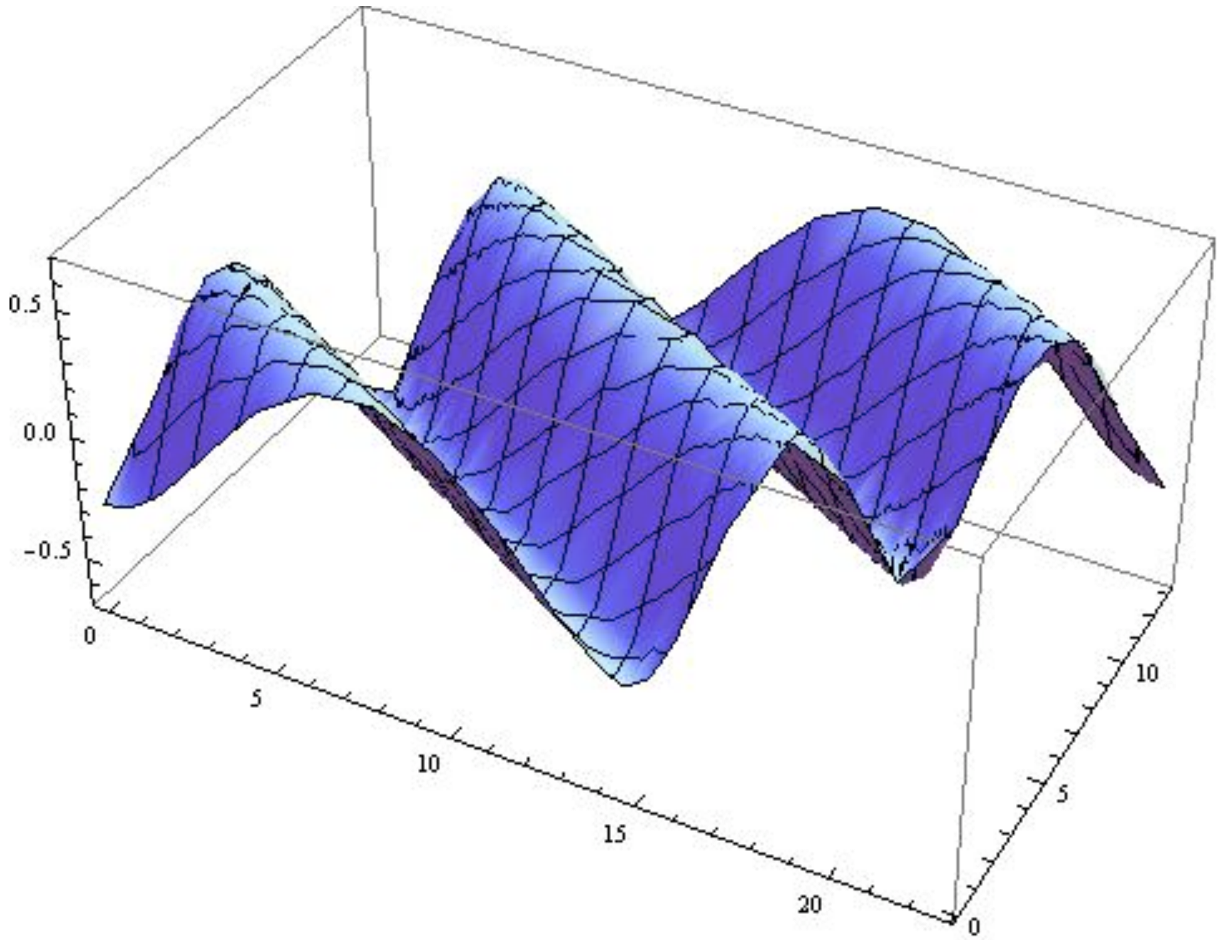


D1-brane, winding (2,-1)

E=4.33

E_{exp}=4.16

W=4.28

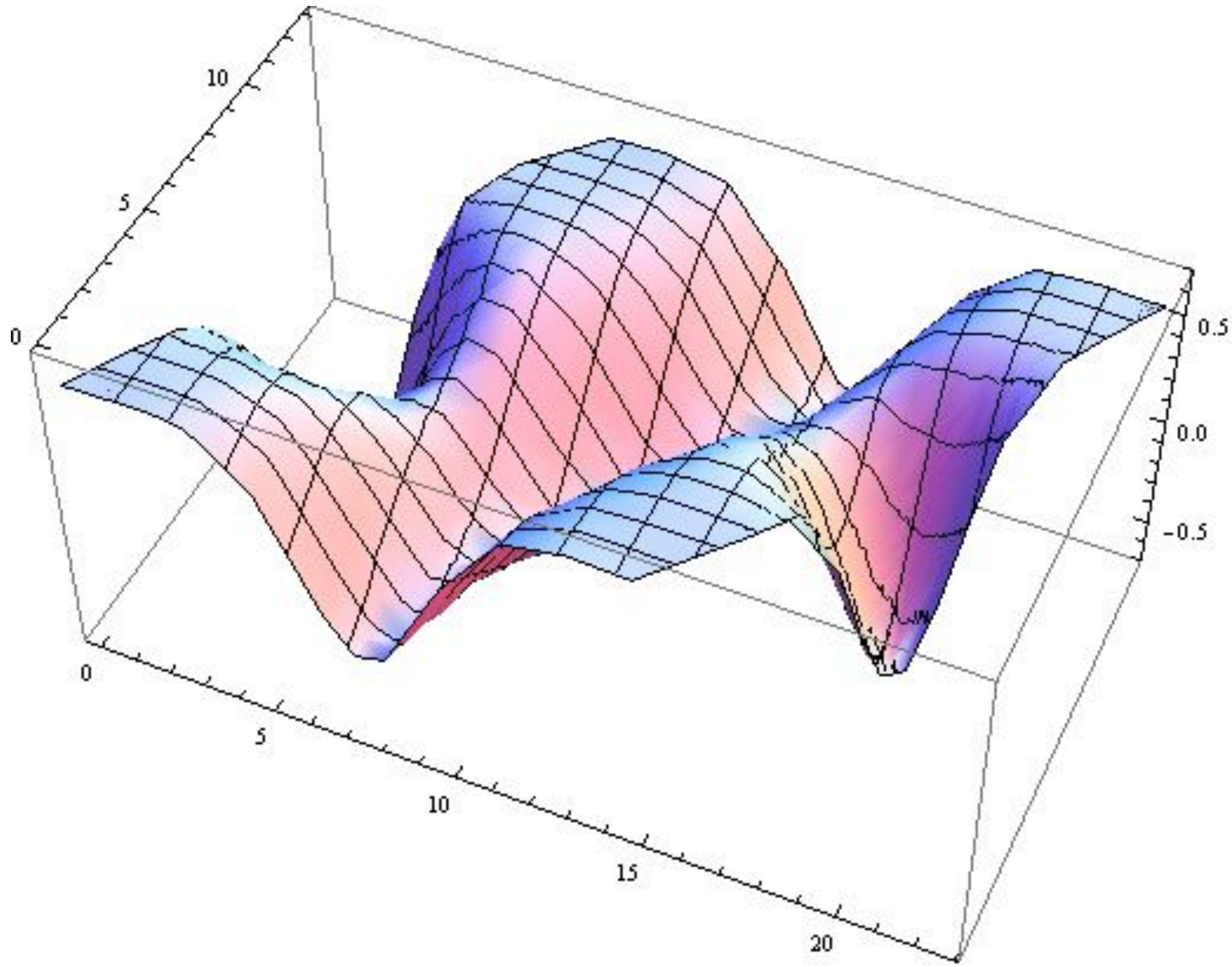


D0-brane and D1-brane

E=3.53

E_{exp}=3.4

W=3.52

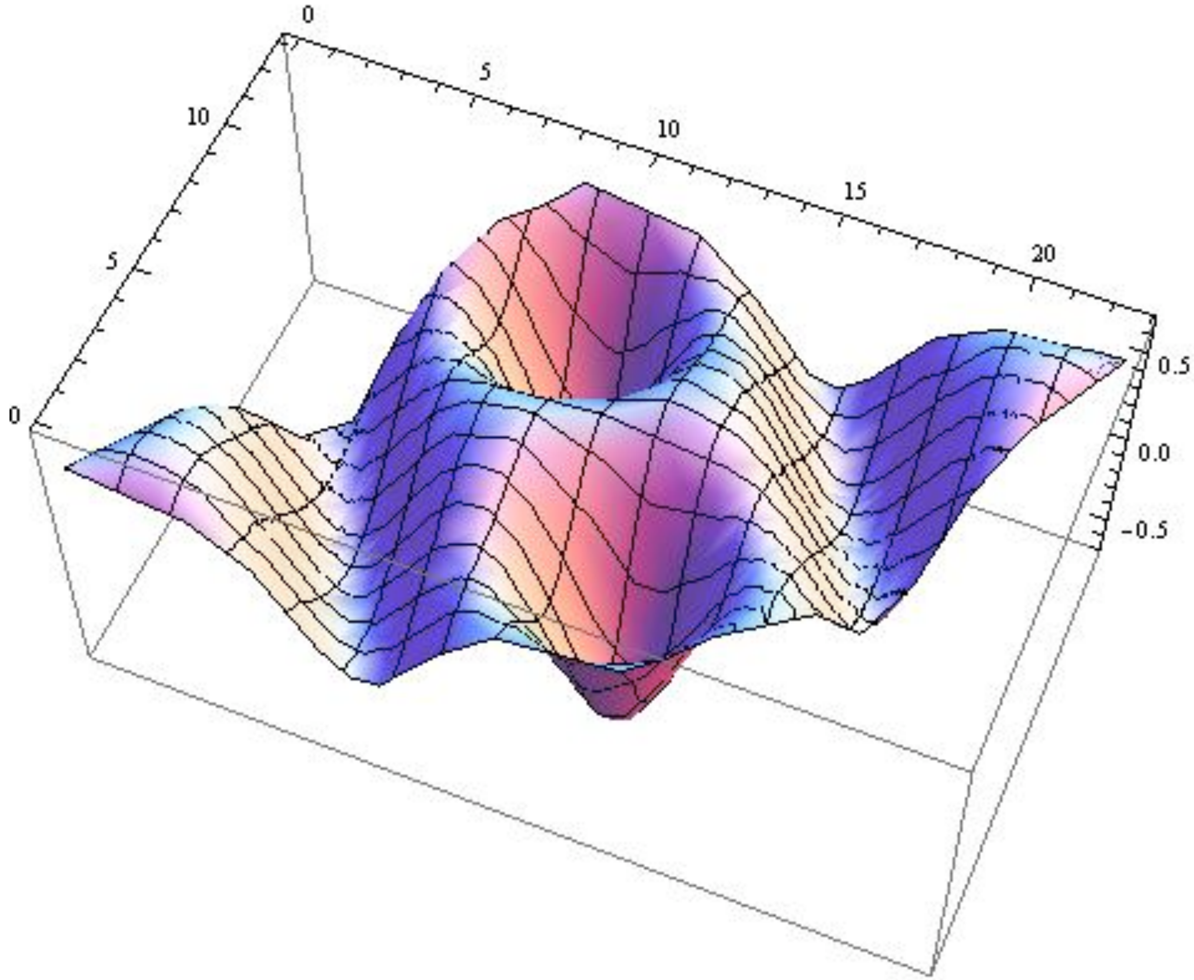


2 D0-branes and D1-brane

$E=4.60$

$E_{exp}=4.4$

$W=4.45$

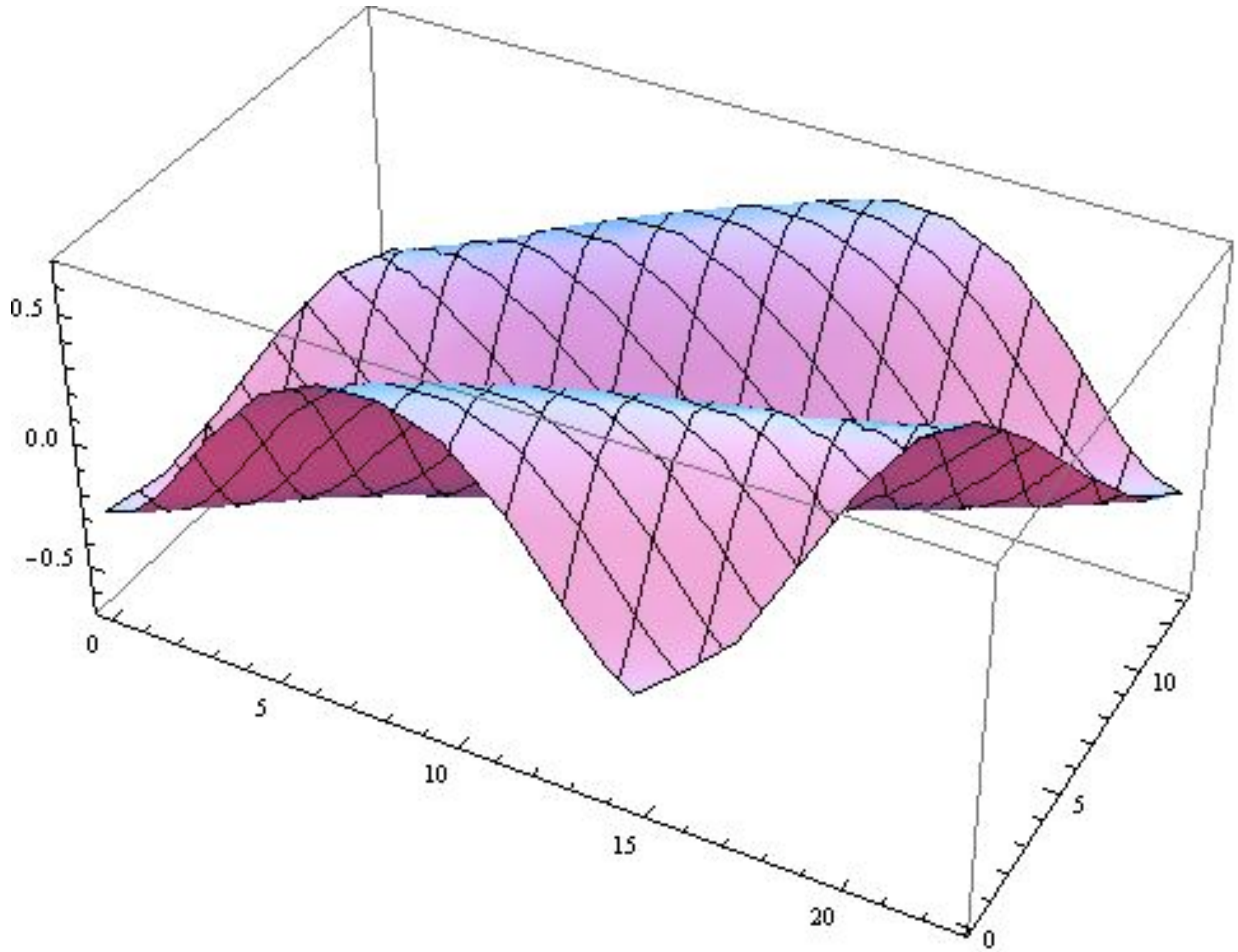


deformed D1-brane ?

E=4.34

E_{exp}=4.16 ?

W=4.29

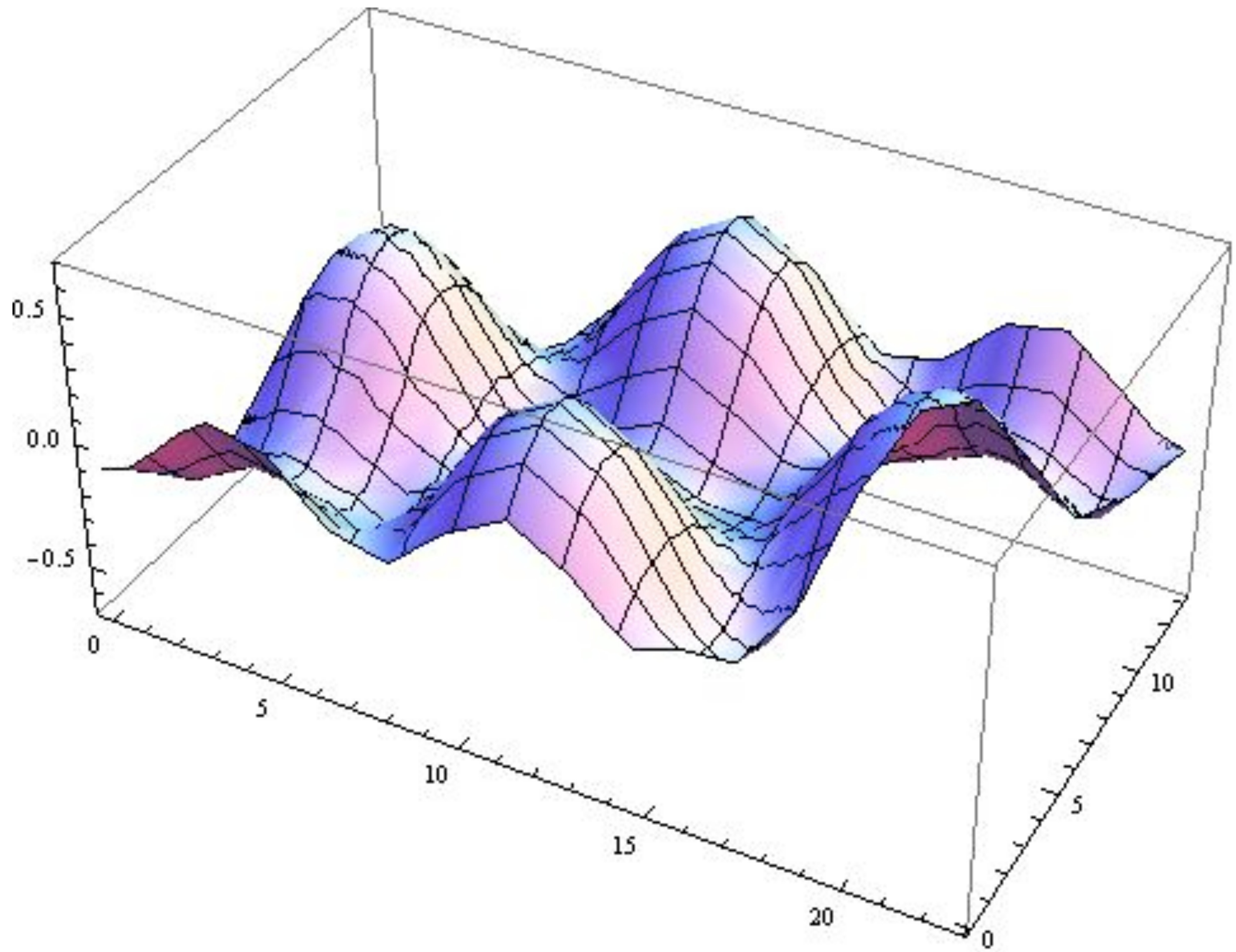


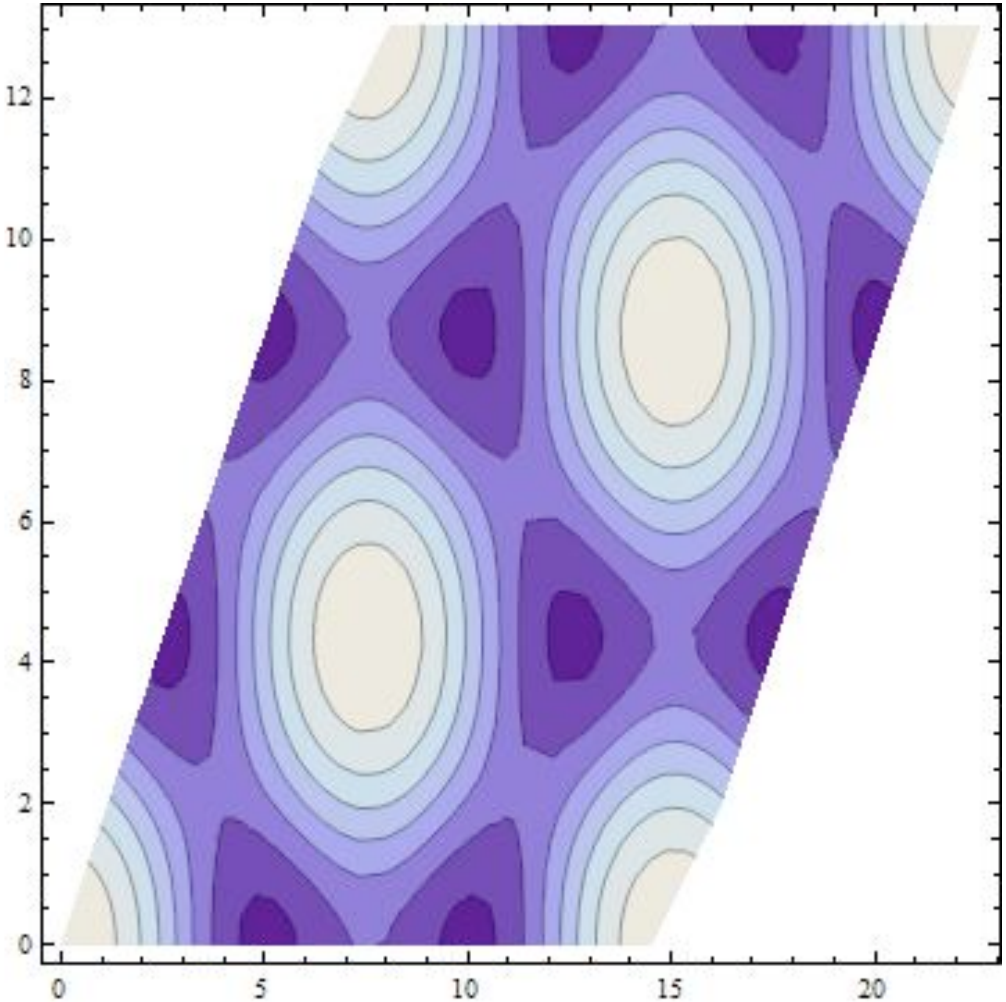
???

E=4.74

E_{exp}=?

W=4.72




0
2
4
6
8
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12
0
5
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15
20