On Lumps from RG flows

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This is an informal transcription of my blackboard talk. It is based on the paper 1105.6057 with Ted Erler. Previous material is given in 1009.4158, in collaboration with Loriano Bonora and Driba Tolla.

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1 Introduction

The idea I want to pursue is to associate a given worldsheet boundary RG flow interpolating from a reference $BCFT_0$ to a target $BCFT^*$, with a classical solution of OSFT describing its IR fixed point, $BCFT^*$.
A world-sheet boundary RG flow can be generated by deforming the original worldsheet action with the integration of a boundary *relevant* operator, which breaks conformal invariance and thus triggers the theory to a new conformal fixed point.

$$S(u) = S_{Disk}^{(BCFT_0)} + \int_{\partial Disk} ds\phi_u(s), \tag{1.1}$$

where u is an implicit definition of the boundary coupling/RG scale.
Here we concentrate on the simplest possible situation, when the boundary interaction is finite without the need of renormalization of contact term divergences.
We stick to boundary interactions (**condition 1**) such that

$$e^{-\int_a^b \phi(s)ds} \tag{1.2}$$

is finite by itself. In conformal perturbation theory, this is an assumption about the finiteness of the integrated n-point functions of $\phi(s)$, along the boundary segment (a, b)

$$\left(\int_a^b\right)^n ds_1....ds_n \left\langle \phi(s_1)...\phi(s_n)\right\rangle_{C_t}. \tag{1.3}$$

This typically happens when the collisions between the ϕ's is less divergent than a simple pole so that the integration will pass through without producing infinities.
Another condition (**condition 2**) is the BRST variation of ϕ

$$Q\phi = \partial(c\phi) - \phi'\partial c. \tag{1.4}$$

The new generated operator ϕ' can be understood as the deviation of ϕ to be a marginal operator, and thus the deviation of producing a conformal boundary interaction. In the X-BCFT describing a flat target space, this is realized by any 'tachyon profile' of the form

$$\phi =: f(X) : . \tag{1.5}$$

The physical content of the solution is encoded in the RG-condition (**condition 3**): *ϕ triggers an RG flow from the reference conformal field theory,* BCFT$_0$*, to a target* *boundary conformal field theory,* BCFT*. For string field theory purposes, this means that a ϕ boundary interaction in correlation functions on a very large cylinder imposes BCFT* boundary conditions, while on a very small cylinder it imposes BCFT$_0$ boundary conditions. Explicitly,

$$\lim_{L\to\infty}\left\langle \exp\left[-\int_0^L ds\,\phi(s)\right]\,L\circ\mathcal{O}\right\rangle_{C_L}^{\mathrm{BCFT}_0} = \lim_{L\to\infty}\big\langle L\circ\mathcal{O}\big\rangle_{C_L}^{\mathrm{BCFT}^*},$$

and

$$\lim_{L\to 0}\left\langle \exp\left[-\int_0^L ds\,\phi(s)\right]\,L\circ\mathcal{O}\right\rangle_{C_L}^{\mathrm{BCFT}_0} = \lim_{L\to 0}\big\langle L\circ\mathcal{O}\big\rangle_{C_L}^{\mathrm{BCFT}_0}. \tag{1.6}$$

where $\langle\cdot\rangle_{C_L}^{\mathrm{BCFT}}$ is a correlator on a cylinder of circumference L in the corresponding BCFT, and $L\circ\mathcal{O}$ is a scale transformation of an arbitrary bulk operator $\mathcal{O}$ under $z\to Lz$. Scaling (1.6) to a canonical cylinder of circumference 1, these conditions can be equivalently stated:

$$\lim_{u\to\infty}\left\langle \exp\left[-\int_0^1 ds\,\phi_u(s)\right]\,\mathcal{O}\right\rangle_{C_1}^{\mathrm{BCFT}_0} = \big\langle\mathcal{O}\big\rangle_{C_1}^{\mathrm{BCFT}^*},$$

and

$$\lim_{u\to 0}\left\langle \exp\left[-\int_0^1 ds\,\phi_u(s)\right]\,\mathcal{O}\right\rangle_{C_1}^{\mathrm{BCFT}_0} = \big\langle\mathcal{O}\big\rangle_{C_1}^{\mathrm{BCFT}_0}, \tag{1.7}$$

where we introduce the operator

$$\phi_u(s) = u(u^{-1}\circ\phi(us)). \tag{1.8}$$

This gives

$$\phi_u' = u\partial_u\phi_u. \tag{1.9}$$

Once a ϕ satisfying the above condition is given, also the corresponding ϕ_u will satisfy the same conditions, and this gives a one parameter family of gauge equivalent choices, related by the midpoint preserving reparametrization generated by $L^- = 1/2(\mathcal{L}_0 - \mathcal{L}_0^*)$. The parameter u (equivalently L) can be interpreted as the RG coupling (or time). In general, $\phi(s)$ will be a sum of different matter operators. To trigger a flow to BCFT* as described in (1.6), the coupling constants multiplying each component matter operator must be precisely chosen.

To appreciate this point it is worth noticing that after ϕ is tuned to satisfy condition 3, any slight modification of it can drastically change the nature of the IR fixed point.

The simplest example of such phenomenon (which will also turn out very useful in the computation of OSFT observables) is given by replacing a tuned ϕ with $\phi+\epsilon$. Then, for $\epsilon>0$, condition 3 immediately implies that all IR quantities will be exponentially suppressed by $\lim_{L\to\infty}\exp[-\epsilon L]=0$. In other words, if ϕ brings to $BCFT^*$, $\phi+\epsilon$ brings to a BCFT with an exponentially suppressed boundary state: the tachyon vacuum. For $\epsilon<0$ we encounter exponentially divergent IR correlators, which we are not able to give a sensible meaning.

At least two such exact RG-flows are known. The first is the cosine deformation given by

$$\phi_u(s)=u\left[-\frac{1}{u^{1/R^2}}:\cos\left(\frac{X(s)}{R}\right):+A(R)\right],, \tag{1.10}$$

with $A(R)$ determined in 1009.4158. This describes the condensation of a brane wrapping a circle of radius R (UV fixed point) to a codimension 1 brane living at the minimum of the ϕ_u profile, that is $x=\pi R$. In order to generate a finite boundary interaction it must be that the weight of ϕ is not too close to 1 (which is where the deformation becomes exactly marginal). In particular we must have $R>\sqrt{2}$.

A second example is the Witten deformation, which describes a codimension one brane along a noncompact direction:

$$\phi_u(s)=u\left[\frac{1}{2}:X(s)^2:+\gamma-1+\ln(2\pi u)\right]. \tag{1.11}$$

where γ is the Euler constant. Unlike the cosine deformation, the Witten deformation leads to a Gaussian worldsheet theory, and Green's functions can be computed exactly.

While it is useful to have these two explicit examples of ϕ in mind, all OSFT gauge invariants only depend on the *universal* behaviour of the ϕ–boundary interaction near the IR fixed point, postulated in the third condition.

1.1 Deformed wedge states

One simple way to include the ϕ boundary interaction in a SFT setting is to enlarge the well-known K,B,c algebra with the identity based insertion

$$\phi \;=\; \phi(1/2)I, \tag{1.12}$$

$$\phi' \;=\; \phi'(1/2)I, \tag{1.13}$$

where I is the identity string field and local insertions are defined directly on the cylinder coordinate frame.
From conditions 1 and 2 we get the algebraic and differential star algebra relations

$$(c\phi)^2 = (c\phi)(c\phi') = (c\phi')(c\phi') = 0 \tag{1.14}$$

$$Q(c\phi) = \phi' c\partial c. \tag{1.15}$$

Just as a strip of world-sheet of width L corresponds to the string-field e^{-LK} (a wedge state), the same strip with a boundary deformation given by $e^{-\int \phi}$ (a *deformed* wedge state) corresponds to

$$\tilde{\Omega}^L = e^{-L(K+\phi)}. \tag{1.16}$$

An important quantity is given by the trace of a deformed wedge state (with an understood ghost insertion to saturate the central charge and the ghost number)

$$g(L) = Tr[\tilde{\Omega}^L] = \left\langle e^{-\int_0^L ds\phi(s)} \right\rangle_{C_L}. \tag{1.17}$$

This quantity (the semi-infinite cylinder partition function) is just the famous 'g-function' of Affleck and Ludwig and its logarithm is the so-called *boundary entropy*. Condition 3 states that we have

$$\lim_{L\to\infty} g(L) = \mathrm{Tr}[\tilde{\Omega}^\infty] = \langle 0|0\rangle^{BCFT^*},\ (IR) \tag{1.18}$$

$$\lim_{L\to 0} g(L) = \mathrm{Tr}[1] = \langle 0|0\rangle^{BCFT_0},\ (UV). \tag{1.19}$$

In other words the deformed sliver $\tilde{\Omega}^\infty$ is associated to the IR fixed point ($BCFT^*$), while the identity string field $I = \tilde{\Omega}^0 = \Omega^0$ is associated with the UV ($BCFT_0$).
The g-theorem/conjecture states that $g(L)$ is a positive decreasing function of L. This essentially states that the system looses (space-time) energy by going from the UV to the IR. In appropriate units the vacuum energy of $BCFT_0$ is given by $\frac{1}{2\pi^2}g(0)$, and the vacuum energy of $BCFT^*$ is given by $\frac{1}{2\pi^2}g(\infty)$.
We point out the following important property

$$Tr[\tilde{\Omega}^L\phi'] = Tr[\tilde{\Omega}^L(K+\phi)] = -\frac{d}{dL}g(L). \tag{1.20}$$

Notice that, because $g(\infty)$ is finite

$$\lim_{L\to\infty} LTr[\tilde{\Omega}^L\phi'] = -\lim_{L\to\infty} L\frac{d}{dL}g(L) = 0. \tag{1.21}$$

The one-point function of ϕ' on a very large (deformed) cylinder goes to zero *faster* than the inverse lenght of the cylinder. These are the only (universal) informations which are needed to reproduce the correct gauge–invariant observables.

2 The solution and its problem

The solution which was proposed one year ago by myself and Bonora and Tolla in 1009.4158 (BMT from now on) is given by

$$\Phi = c\phi - \frac{B}{K+\phi}\phi' c\partial c = c\phi - B\int_0^\infty dt\,\tilde{\Omega}^t \phi' c\partial c. \tag{2.1}$$

Leaving for later the check of the equation of motion, let's instead concentrate on the simplest gauge invariant observable which can be computed, namely the Ellwood invariant (or the closed string overlap). In 1009.4158 it was shown that

$$\mathrm{Tr}[\mathcal{V}\Phi] = -\mathcal{A}_0(\mathcal{V}) + \mathcal{A}^*(\mathcal{V}), \tag{2.2}$$

where $\mathcal{A}_0(\mathcal{V})$ is the disk amplitude in BCFT_0 with one on-shell closed string insertion $\mathcal{V} = c\tilde{c}\mathcal{V}^{\mathrm{m}}$, $\mathcal{A}^*(\mathcal{V})$ is the same quantity in BCFT^*, and $\mathrm{Tr}[\mathcal{V}\cdot]$ is the 1-string vertex with a midpoint insertion of $\mathcal{V}$. The very reason behind this result has to be traced back to the following simple trace in the matter sector

$$\begin{aligned} Tr[\frac{1}{K+\phi}\phi'] &= \int_0^\infty dL\, Tr[\tilde{\Omega}^L\phi'] = \int_0^\infty dL\, Tr[\tilde{\Omega}^L(K+\phi)] \\ &= -\int_0^\infty dL\, \frac{d}{dL}g(L) = g(\infty) - g(0), \end{aligned} \tag{2.3}$$

where (1.20) has been used.

Thus the main building block of the solution, namely $\frac{1}{K+\phi}\phi'$, is correctly producing the desired shift from the UV to the IR.

However, looking carefully, we realize that this shift is happening *because* the Schwinger representation of the formal string field $\frac{1}{K+\phi}$ is failing in inverting $(K+\phi)$

$$\int_0^\infty dL\,\tilde{\Omega}^L(K+\phi) = \int_0^\infty dL\, e^{-L(K+\phi)}(K+\phi) = -\int_0^\infty dL\,\frac{d}{dL}\tilde{\Omega}^L = 1 - \tilde{\Omega}^\infty. \tag{2.4}$$

This brings problems with the equation of motion.

3 Equation of motion

To clearly see what is the intrinsic problem with the equation of motion it is convenient to realize that the BMT solution has the generic structure

$$\Phi = c\phi - A\phi' c\partial c, \tag{3.1}$$

where A is some ghost number -1 string field. Let's see how a string field of this form can satisfy the equation of motion. Compute

$$Q\Phi = (1 - QA)\phi' c\partial c, \tag{3.2}$$

and (using $(c\phi')(c\phi) = (c\phi)(c\phi) = 0$)

$$\Phi^2 = -\Phi A\phi' c\partial c = -[\Phi, A]\phi' c\partial c. \tag{3.3}$$

Notice that we took the freedom of adding zero in the form $((\phi' c\partial c)^2 = 0)$

$$0 = A\Phi\phi' c\partial c.$$

Putting everything together we find

$$Q\Phi + \Phi^2 = (1 - Q_\Phi A)\phi' c\partial c, \tag{3.4}$$

where $Q_\Phi = Q + ad_\Phi$ is the shifted kinetic operator around the string field Φ. One obvious way to solve the equation of motion is asking for $\phi' = 0$ (that is ϕ is a marginal operator), but this just gives an identity based solution for a (regular) marginal deformation. So that's not what we are searching for. The other way is to impose

$$Q_\Phi A = 1. \tag{3.5}$$

But if this is realized then A is an homotopy operator and Φ is the tachyon vacuum (i.e. the IR fixed point is trivial).
Plugging the BMT definition

$$A = B\int_0^\infty dt\, \tilde{\Omega}^t$$

we find instead

$$Q_\Phi A = 1 - \tilde{\Omega}^\infty. \tag{3.6}$$

So the cohomology of Φ is not trivialized (in exactly the same way as the cohomology of Q is not trivialized by B/K) and, *at the same time*, the equation of motion is violated.

$$Q\Phi + \Phi^2 = \tilde{\Omega}^\infty \phi' c\partial c \equiv \Gamma \neq 0. \tag{3.7}$$

There is another interesting way of appreciating the problem with the equation of motion. To this end we write the BMT solution as

$$\Phi = c\phi + BP_\phi c, \tag{3.8}$$

and try to determine the ghost number 1 string field P_ϕ by solving the equation of motion. It is easy to realize that the equation of motion boils down to the following linear equation for P_ϕ

$$(K+\phi)P_\phi = \phi' \partial c. \tag{3.9}$$

This innocent looking equation is where all the troubles come in. The left hand side has indeed a left kernel given by the deformed sliver[2]

$$\tilde{\Omega}^\infty (K+\phi)P_\phi = 0, \tag{3.10}$$

Thus the above linear equation is subject to the obvious integrability condition (see also the talk by Ted Erler for a more general discussion)

$$\tilde{\Omega}^\infty \phi' \partial c =?= 0, \tag{3.11}$$

which, as we will see in a moment, is not true in the Fock space. Again, it is important to see that the Schwinger representation

$$P_\phi = \int_0^\infty dt\, \tilde{\Omega}^t \phi' \partial c$$

is indeed realizing

$$(K+\phi)P_\phi = (1-\tilde{\Omega}^\infty)\phi' \partial c. \tag{3.12}$$

And again we find

$$Q\Phi + \Phi^2 = \tilde{\Omega}^\infty \phi' c \partial c. \tag{3.13}$$

Notice that the finiteness of the deformed sliver $\tilde{\Omega}^\infty$ is behaving as a topological obstruction which, on the one hand, prevents from flowing to the tachyon vacuum and, on the other hand, prevents from solving the equation of motion.
It is important to realize that this is precisely the point where all other known regular solutions don't fail. An instructive example is given by the well known string field

$$P = \frac{K}{1-\Omega}$$

[2]assuming P_ϕ is not plagued by associativity anomalies in the star product

entering the Schnabl solution. This string field can be defined as the solution to the linear equation

$$(1-\Omega)P = K \tag{3.14}$$

Again, since the lhs is left–killed by the sliver ($\Omega^\infty - \Omega^{\infty+1} = 0$), we get the integrability condition $\Omega^\infty K = 0$, which is luckily satisfied in a strong enough sense. In particular the above equation can be solved order by order in powers of K, which gives the $\mathcal{L}_0$ expansion of the Schnabl solution. Similar non trivial integrability conditions are satisfied by all known regular solutions which can be written inside a minimal extension of the universal K, B, c algebra.
A quick inspection into

$$\Gamma = \tilde{\Omega}^\infty \phi' c\partial c$$

shows that it is not vanishing because ϕ' sits at the edge of the sliver, where it can have contractions with nearby operators. As shown in 1105.6057 this gives finite overlaps with Fock-space states and, more seriously, finite overlap with the solution itself, raising serious doubts about the possibility of reproducing the correct value for the action.

4 Regularization

To continue with explicit computations it is convenient (but not compulsory[3]) to regularize the divergent Schwinger integral

$$\int_0^\infty dt\, \tilde{\Omega}^t.$$

The obvious way to do so is to introduce a damping factor (for example an hard cut-off in the IR region). There is however a better choice, which is taylor-made to the simple form of the solution.

$$\int_0^\infty dt\, \tilde{\Omega}^t = \lim_{\epsilon\to 0} \int_0^\infty dt\, e^{-\epsilon t}\tilde{\Omega}^t = \lim_{\epsilon \to 0} \frac{1}{K+\phi+\epsilon}. \tag{4.1}$$

[3]This nice feature is a consequence of the fact that, at least in the explicit case of the Witten deformation, $\mathrm{Tr}[\Phi^3]$ is given by an absolutely convergent integral in the 3 Schwinger parameters, which makes the value of the integral independent of the way the three cutoffs are removed. Due to universal behavior near the IR we expect this to continue to hold for the cosine deformation as well.

This, like any other way of representing $1/(K+\phi)$ as a limit of states in the deformed wedge algebra, gives a geometric definition of Φ, through the regularized state

$$\begin{aligned} \Phi(\epsilon) &= c\phi - \frac{B}{K+\phi+\epsilon}\phi' c\partial c, &(4.2)\\ \Phi &= \lim_{\epsilon\to 0^+}\Phi(\epsilon). &(4.3)\end{aligned}$$

This regularization allows to represent the BMT solution as the sum of a tachyon vacuum contribution plus a 'phantom' term in the following way

$$\begin{aligned} \Phi(\epsilon) &= \Psi_{tv}(\epsilon)+\Delta(\epsilon) &(4.4)\\ \Psi_{tv}(\epsilon) &= c(\phi+\epsilon) - \frac{B}{K+\phi+\epsilon}(\phi'+\epsilon)c\partial c &(4.5)\\ \Delta(\epsilon) &= -\epsilon c + B\frac{\epsilon}{K+\phi+\epsilon}c\partial c. &(4.6)\end{aligned}$$

Going back to the assumptions about the RG flow, we notice that $\Psi_{tv}(\epsilon)$ is just the BMT solution with $\phi\to\phi+\epsilon$, and we already observed that this RG flow ends in the tachyon vacuum, because of the exponential suppression of the boundary state. This is indeed captured in the OSFT language by the existence of the regular homotopy field $\frac{B}{K+\phi+\epsilon}$ which genuinely trivialize the cohomology of $\Psi_{tv}(\epsilon)$

$$(Q+ad_{\Psi_{tv}(\epsilon)})\frac{B}{K+\phi+\epsilon} = 1,\ \forall\epsilon>0. \tag{4.7}$$

The phantom term $\Delta(\epsilon)$ is naively vanishing in the $\epsilon\to 0$ limit, however we notice that

$$\lim_{\epsilon\to 0}\frac{\epsilon}{K+\phi+\epsilon} = \tilde{\Omega}^\infty, \tag{4.8}$$

so the 'un-regulated' phantom term is really

$$\Delta = B\tilde{\Omega}^\infty c\partial c, \tag{4.9}$$

and will thus contribute to observables.

Computing the equation of motion we find the same result as before

$$\lim_{\epsilon\to 0}(Q\Phi(\epsilon)+\Phi(\epsilon)^2) = \lim_{\epsilon\to 0}\frac{\epsilon}{K+\phi+\epsilon}\phi' c\partial c = \lim_{\epsilon\to 0}\Gamma(\epsilon) = \Omega^\infty\phi' c\partial c = \Gamma. \tag{4.10}$$

As a last remark, we notice the non trivial relation

$$\Delta(\epsilon)\Phi(\epsilon) = \Gamma(\epsilon), \tag{4.11}$$

which expresses the anomaly in the equation of motion as the product of the phantom with the solution itself. We are not aware of any regularization or definition of the solution which solves the equation of motion in a strong enough sense and doesn't change the physics in the IR.

5 Energy

Since the equation of motion are violated, we compute the energy by evaluating the full gauge invariant action. This gives a surprise. Taking into account the anomaly in the equation of motion and using the decomposition (4.4) and the nontrivial relation (4.11) we get

$$\begin{aligned} E[\Phi(\epsilon)] &= \mathrm{Tr}\left[\frac{1}{2}\Phi(\epsilon)Q\Phi(\epsilon)+\frac{1}{3}\Phi(\epsilon^3)\right] \\ &= \mathrm{Tr}\left[\frac{1}{6}\Psi_{\rm tv}(\epsilon)^3\right]+\mathrm{Tr}\left[\frac{1}{6}\Delta(\epsilon)^3+\frac{1}{2}\Gamma(\epsilon)\Phi(\epsilon)-\frac{1}{2}\Delta(\epsilon)\Gamma(\epsilon)-\frac{1}{2}\Phi(\epsilon)\Gamma(\epsilon)\right] \\ &= Tr\left[\frac{1}{6}\Psi_{\rm tv}(\epsilon)^3\right]+\mathrm{Tr}\left[\frac{1}{6}\Delta(\epsilon)^3-\frac{1}{2}\Delta(\epsilon)\Gamma(\epsilon)\right]. \end{aligned} \tag{5.1}$$

The anomaly contracted with the solution is not vanishing. For the Witten deformation we explicitly computed it

$$\lim_{\epsilon\to 0}\mathrm{Tr}[\Phi(\epsilon)\Gamma(\epsilon)]=-\frac{g(\infty)}{\pi^2}w, \tag{5.2}$$

where w is the value of the integral

$$w=\int_0^\infty dx\, x(\cos(x)\,\mathrm{ci}(x)+\sin(x)\mathrm{si}(x))^2\simeq 0.36685. \tag{5.3}$$

However, notice that in(5.1) the terms with the anomaly contracted with the solution cancel between the kinetic and cubic term of the action. The anomaly is still present, but only contracted with the phantom term (which doesn't contain insertions of ϕ'). The sliver like structure of this correlator will thus give a contribution proportional to the 1 point function of ϕ' on a very large cylinder which, as a consequence of the g–theorem vanishes. Explicitly (taking into account the ghost correlator which contributes a linear divergence) we find the $BSFT$–looking result

$$\mathrm{Tr}\left[\frac{1}{2}\Delta(\epsilon)\Gamma(\epsilon)\right]=\frac{1}{2\pi^2}\lim_{L\to\infty}L\frac{d}{dL}g(L)=0. \tag{5.4}$$

This is the 'physical' way the anomaly effectively vanishes inside the gauge invariant action.

The remaining term $\mathrm{Tr}[\Delta^3]$ (modulo ghost factors) is the trace of the deformed sliver state, with *no* insertion of ϕ'. Thus the matter part will contribute $g(\infty)$. Explicitly, in the $\epsilon\to 0$ limit we find

$$\lim_{\epsilon\to 0}(E[\Phi(\epsilon)]-E[\Psi_{\rm tv}(\epsilon)])=\lim_{\epsilon\to 0}\mathrm{Tr}\left[\frac{1}{6}\Delta(\epsilon)^3\right]=\frac{g(\infty)}{2\pi^2}, \tag{5.5}$$

or equivalently

$$\lim_{\epsilon\to 0} E[\Phi(\epsilon)] = \frac{1}{2\pi^2}(g(\infty) - g(0)), \tag{5.6}$$

which explicitly reproduces the shift between the vacuum energy of $BCFT_0$ and $BCFT^*$.

Notice that, in the $\epsilon \to 0$ limit, $\Delta(\epsilon)$ can be alternatively interpreted as the lump solution around the tachyon vacuum $\Psi_{tv}(\epsilon)$. The fact that only $\mathrm{Tr}[\Delta\Gamma] = 0$ enters the gauge invariant action explains why the authors of 1105.5926 found the correct energy by expanding the solution around $\Psi_{tv}(\epsilon)$ and computing just the cubic term in the action.

6 Conclusions

I end up listing some more comments and further directions.

6.1 Cohomology

The way the gauge invariant action works suggests that there is a whole class of sliver like states (which the phantom term belongs to) against which the anomaly is vanishing.

$$\lim_{\epsilon\to 0} \mathrm{Tr}[\Gamma(\epsilon)\Pi(\epsilon)] = 0.$$

Inside this family the shifted BRST operator is nilpotent and defines a cohomology

$$Q^2_{\Phi(\epsilon)} = ad_{Q\Phi(\epsilon)+\Phi(\epsilon)^2} = ad_{\Gamma(\epsilon)} \to 0.$$

In fact, as we showed in 1105.6057, there is a systematic way to represent Fock states of the target $BCFT^*$ with projector like representatives in $BCFT_0$. When restricted to these states the BMT kinetic operator becomes indistinguishable from the BRST operator of $BCFT^*$. In particular we considered fluctuations of the form

$$\Pi_i(\epsilon) = \tilde{\Omega}^{\frac{1}{2\epsilon}} \pi_i(\epsilon) \tilde{\Omega}^{\frac{1}{2\epsilon}}.$$

These projector like states are chosen to flow to $BCFT^*$ Fock states

$$\Pi_i^* = \Omega_*^{\frac{1}{2}} \pi_i^* \Omega_*^{\frac{1}{2}}$$

in the following sense

$$\lim_{\epsilon\to 0} \langle \Pi_i(\epsilon), \Pi_j(\epsilon)\rangle^{BCFT_0} = \langle \Pi_i, \Pi_j\rangle^{BCFT^*}. \tag{6.1}$$

Using these states we were able to rewrite the expanded OSFT action around $\Phi(\epsilon)$ as the action directly formulated in $BCFT^*$.

$$\lim_{\epsilon\to 0} S^{(BCFT_0)}(\Phi(\epsilon)+\Pi(\epsilon)) = \frac{1}{2\pi^2}(g(0)-g(\infty)) + S^{(BCFT^*)}(\Pi^*). \tag{6.2}$$

Thus it is possible to find cohomology representatives in a subspace where the anomaly vanishes. Notice however that generic states will feel the non nilpotency of Q_Φ and they will experience a tadpole sourced by the anomaly, since Φ is off-shell from the perspective of the UV degrees of freedom.

6.2 Better regularizations?

Assuming the existence of the lump solution (which was numerically found in Siegel gauge by Moeller, Sen and Zwiebach) there is the possibility that the BMT solution lives in a singular point of a gauge orbit of regular solutions. In this case a correct regularization should displace the BMT solution from this singular point to a generic point in the orbit. It would be desirable to develop some new method to systematically build regular solutions starting from off-shell configurations.

6.3 Relation with Multibranes

It is interesting to notice that the recently proposed multibrane solutions by Murata and Schnabl (see also Masuda and Okawa) encounter similar problems in merging together the correct observables with the equation of motion. It would be very interesting to analyze the similarities (and the differences) between these two class of solutions.

6.4 Relation with bcc operators

It would be also very interesting to understand the relation between the way we go the the IR (that is through an RG flow) and the description of the same $BCFT^*$ through the use of boundary condition changing operators, as suggested by Kiermaier, Okawa and Soler and, more recently, by Noumi and Okawa.

THANK YOU.

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