# Comments on multi-brane solutions in open string field theory

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## Witten's bosonic, open, cubic string field theory

Known exact solutions;

- tachyon vacuum
- marginal deformations (regular, singular)
- relevant deformations
  - lower dimentional D-brane (on going?)

Question;

| can we describe multiple D25-brane backgrounds in CSFT ? |
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This is a hard question, but

Murata-Schnabl's work (2010) suggests the existence of multibrane solutions.

## Theme of this talk

We calculate the energy of the specific solution:

$$\Psi = \frac{1}{K} c \frac{K^2}{K-1} c$$

We don't know the definition of $1/K$. We tried several approach.

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Results:

- Schwinger parametrization is not good.
  - energy value is strange.
  - eom contracted with itself is broken.

- Some <u>prescriptions</u> go well.
  - $\Psi$ reproduces two-brane energy.
  - eom contracted with itself is satisfied. $\square$

## Plan to talk

1. Brief review of CSFT $\sim$ Murata-Schnabl's energy formula

2. Some comments on the property of $K$ or $1/K$

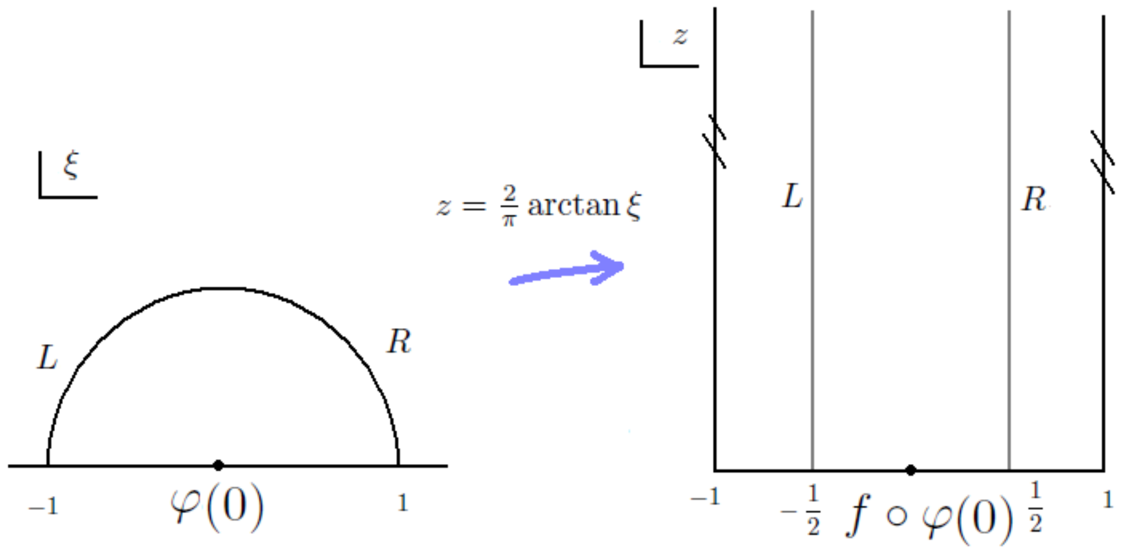
3. Energy calculation

## 1. CSFT ∼multibrane, Murata-Schnabl's formula

String field

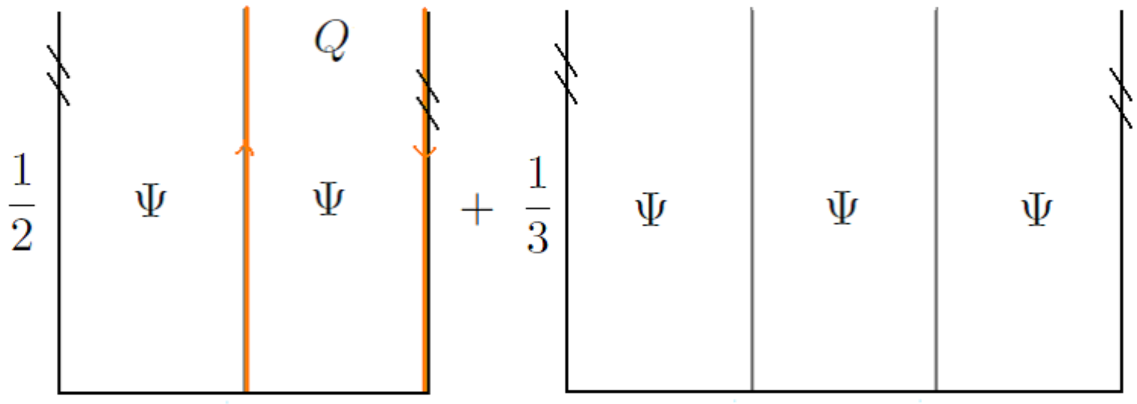
$$\Psi = t(x)c_1|0\rangle + A_\mu(x)a^\mu_{-1}c_1|0\rangle \dots$$

It is convinient to use $z = \frac{2}{\pi}\arctan\xi$ coordinates,



CSFT action;

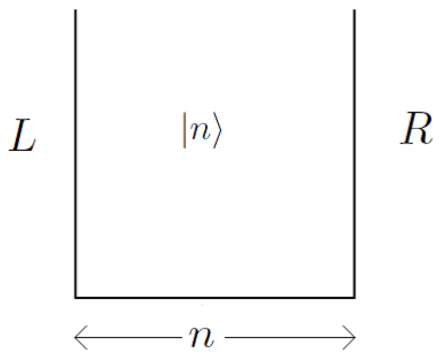
$$S[\Psi] = \frac{1}{2}\langle \Psi, Q\Psi \rangle + \frac{1}{3}\langle \Psi, \Psi * \Psi \rangle$$
$$= \frac{1}{2}\mathrm{tr}[\Psi Q \Psi] + \frac{1}{3}\mathrm{tr}[\Psi\Psi\Psi]$$



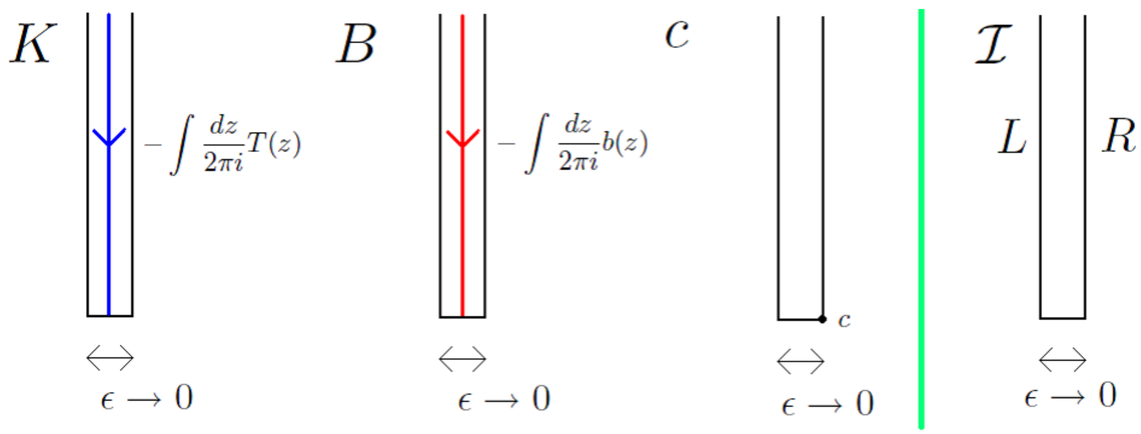

eom

$$Q\Psi + \Psi * \Psi = 0$$

wedge state $|n\rangle$




$KBc$ string field




wedge stete can be written by $K$.

$$|n\rangle = e^{nK}$$

$KBc$ algebra

e.g.

$$B * c + c * B = \mathcal{I}$$

we write this as follows;

$$\{B, c\} = Bc + cB = 1$$

similary,

$$[\,K,\ B\,] = 0\,, \qquad \{\,B,\ c\,\} = 1\,, \qquad B^2 = 0\,, \qquad c^2 = 0\,.$$

The action of BRST charge is

$$QB = K\,, \qquad QK = 0\,, \qquad Qc = cKc\,.$$

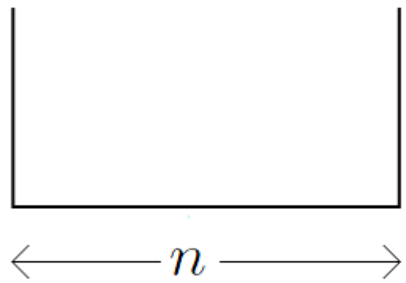
Summary $KBc$

$$[\,K,\,B\,]=0\,,\qquad \{\,B,\,c\,\}=1\,,\qquad B^2=0\,,\qquad c^2=0\,,$$

$$QB=K\,,\qquad QK=0\,,\qquad Qc=cKc\,.$$

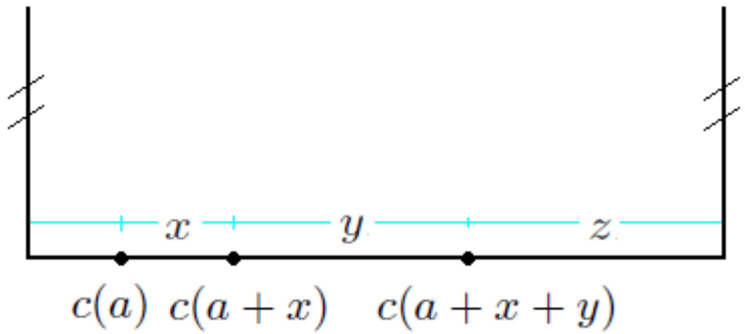
wedge state

$$e^{Kn}=$$




Exapmle of the correlation function

$$\mathrm{tr}[ce^{xK}ce^{yK}ce^{zK}]\ =$$



## Correlation function

$$\mathrm{tr}[ce^{xK}ce^{yK}ce^{zK}] = -\left(\frac{x+y+z}{\pi}\right)^3 \sin\frac{\pi x}{x+y+z}\sin\frac{\pi y}{x+y+z}\sin\frac{\pi z}{x+y+z}$$

$$\begin{aligned}\mathrm{tr}[Bce^{xK}ce^{yK}ce^{zK}ce^{wK}] =& \frac{s^2}{4\pi^3}\left(z\sin\frac{2\pi x}{s} - (y+z)\sin\frac{2\pi(x+y)}{s} + y\sin\frac{2\pi(x+y+z)}{s}\right.\\ &\left. + x\sin\frac{2\pi z}{s} - (x+y)\sin\frac{2\pi(y+z)}{s} + (x+y+z)\sin\frac{2\pi y}{s}\right)\end{aligned}$$

with

$$s = x+y+z+w$$

Using these quantity, we can write down the following formal solution:

$$\Psi = F(K)c\frac{KB}{1-F^2(K)}cF(K)$$

or,

$$\Psi = F^2(K)c\frac{KB}{1-F^2(K)}c$$

when we do not care about reality condition.

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$F(K)$ determines the physical property of the solution.

We also define $G(K) = 1 - F^2(K)$ for later convinience.

# Murata-Schnabl's energy formula(2010)

$$E = \frac{1}{2}\mathrm{tr}[\Psi \mathrm{Q} \Psi] + \frac{1}{3}\mathrm{tr}[\Psi\Psi\Psi] = -\frac{1}{2\pi^2}\lim_{z\to 0} z\frac{G'(z)}{G(z)} \quad (1)$$

i.e.

$$\boxed{\text{If } G(z) \sim z^n \ (z \sim 0), \text{ then } E = \tfrac{1}{2\pi^2}(-n)}$$

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assumptions:

♣ $\Psi$ is the superposition of the wedge states with insertions.

$$\Psi = F^2(K)c\frac{KB}{1-F^2(K)}c = \int dx \int du\, f(x)\, h(u)\, e^{Kx}ce^{Ku}Bc$$

♣ $F^2(z)$, $\frac{z}{1-F^2(z)}$ is holomorphic in $\Re(z) < 0$

(etc...)

Therefore,

| if $G(z) \sim 1/z$ around $z = 0$, then $\Psi$ would be a two-brane solution. |
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So we choose $(F^2(K) = 1/K)$

$$\Psi = \frac{1}{K} c \frac{K^2 B}{K-1} c$$

and calculate the energy.

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$1/K$ is the string tield satisfying $K * 1/K = \mathcal{I}$, but

| it is not clear weather $1/K$ is written as superposition of the wedge states. |
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or rather we do not know weather we can construct $1/K$.

## 2. On the eigenvalues of $K$

$\boxed{K \text{ has zero eigenvalue}}$

Indeed, $e^{K\infty}$ is not 0.

therefore, we cannot take the inverse of $K$ in general.

$\frac{1}{K}$ is regular only if it is placed at appropriate place in a correlation function.

For example,

$$\mathrm{tr}[ce^{Kx}ce^{Ky}ce^{Kz}] = -\left(\frac{x+y+z}{\pi}\right)^3 \sin\frac{\pi x}{x+y+z}\sin\frac{\pi y}{x+y+z}\sin\frac{\pi z}{x+y+z}$$
$$= \int_0^\infty (-yz(y+z)\delta(t) + f(t;y,z))e^{-tx}dt \tag{2}$$

and we see around $e^{Kx}$ zero-mode of $K$ contribute.

Then, we see that $\mathrm{tr}[c\frac{1}{K}e^{Kx}ce^{Ky}ce^{Kz}]$ is singular.

$$
\begin{aligned}
f(t;y,z) &= \sum_{m=1}^{\infty} \frac{2\times(-4)^{m+1}\pi^{2m}}{(2m+3)!(2m+1)!} t^{2m-1}\left\{y^{2m+3}+z^{2m+3}-(y+m)^{2m+3}\right\} e^{-(y+z)t} \\
&\sim -\frac{4}{3}(\pi^2 y^4 z + 2\pi^2 y^3 z^2 + 2\pi^2 y^2 z^3 + \pi^2 y z^4)t + O(t^2)
\end{aligned}
\tag{3}
$$

On the other hand,

$$\mathrm{tr}[ce^{Kx}cK^3e^{Ky}ce^{Kz}] = \int_0^\infty \partial_y^3 f(t;y,z)e^{-tx}dt \tag{4}$$

and we see around $e^{Kx}$ zero-mode of $K$ does not contribute.

Then, $\mathrm{tr}[c\frac{1}{K}e^{Kx}cK^3e^{Ky}ce^{Kz}]$ would be regular.

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In such a case, we expect that the correlation of $\frac{1}{K}e^{K\infty}$ is zero;

$$\mathrm{tr}[c\frac{1}{K}e^{K\infty}cK^3e^{Ky}ce^{Kz}] = 0.$$

# 3. Calculation of the energy

# Schwinger parameterization

We try the Schwinger parameterization:

$$\frac{1}{K} = -\lim_{\Lambda\to\infty}\int_0^{\Lambda} e^{-Kx}dx. \tag{5}$$

Our calcuration is straightfoward.

$$\Psi = \frac{1}{K} c \frac{K^2 B}{K-1} c = - \lim_{\Lambda \to \infty} \int_0^{\Lambda} dx \int_0^{\infty} due^{-u} \partial_u^2 e^{Kx} cBe^{Ku} c$$

and

$$\begin{aligned}
\mathrm{tr}[\Psi Q \Psi] &= \lim_{\Lambda, \Lambda' \to \infty} \int_0^{\Lambda} dx \int_0^{\Lambda'} dy \int_0^{\infty} du \int_0^{\infty} dve^{-u-v} \partial_u^2 \partial_v^2 \mathrm{tr}[e^{Kx} cBe^{Ku} Q(e^{Ky} cBe^{Kv} c)] \\
&= \lim_{\Lambda, \Lambda' \to \infty} \int_0^{\Lambda} dx \int_0^{\Lambda'} dy \int_0^{\infty} du \int_0^{\infty} dve^{-u-v} \partial_u^2 \partial_v^2 N_K(x, y; u, v)
\end{aligned} \tag{6}$$

then evaluate the integral.

$$
\begin{aligned}
&N_K(x,y;u,v) = \mathrm{tr}[e^{xK}ce^{uK}BcQ(e^{yK}ce^{vK}Bc)] \\
&= \frac{1}{2\pi^2}\Bigg\{ -(x+y)s + y(s-x)\cos\frac{2\pi x}{s} + x(s-y)\cos\frac{2\pi y}{s} + uv\cos\frac{2\pi u}{s} + uv\cos\frac{2\pi v}{s} \\
&+ (xy-uv)\cos\frac{2\pi(x+v)}{s} + (xy-uv)\cos\frac{2\pi(y+v)}{s}\Bigg\} \\
&+ \frac{s}{4\pi^3}\Bigg\{ 2y\sin\frac{2\pi x}{s} + 2x\sin\frac{2\pi y}{s} + (s-2v)\sin\frac{2\pi u}{s} + (s-2u)\sin\frac{2\pi v}{s} \\
&+ (x-y+u-v)\sin\frac{2\pi(x+v)}{s} + (x-y-u+v)\sin\frac{2\pi(y+v)}{s}\Bigg\}
\end{aligned}
$$

$$
\sum (\text{ quadratic in } x,\ y,\ u,\ v) \times \sin \text{or} \cos \frac{2\pi x, \text{or something}}{x+y+z+w}
$$

Results

The integral is finite, but

$$\mathrm{tr}[\Psi Q \Psi] = \lim_{\Lambda,\,\Lambda' \to \infty} \mathcal{E}(\Lambda,\,\Lambda') \qquad \leftarrow \text{ the results depends on } \Lambda \text{ and } \Lambda'$$

If we take $\Lambda = \Lambda'$,

$$\mathrm{tr}[\Psi Q \Psi] = -\frac{1}{2} + \frac{1}{\pi^2}.$$

similar calculation for the cubic term;

$$\mathrm{tr}[\Psi\Psi\Psi] = \frac{3}{\pi^2}\left(-\frac{19}{18} - \frac{9\sqrt{3}}{3\pi^2} + \frac{3}{2\pi^2}\right)$$

that is, the equation of motion contracted with itself is broken.

$$\mathrm{tr}[\Psi(Q\Psi + \Psi\Psi)] \neq 0$$

So, we consider the way to omit the contribution of two $\frac{e^{K\infty}}{K}$'s ...

## 1 the use of primitive function

We replace two $1/K$'s with $e^{Kx}$ and $e^{Ky}$ respectively,

$$\mathrm{tr}[\ldots\frac{1}{K}\cdots\frac{1}{K}\ldots]\to\mathrm{tr}[\ldots e^{Kx}\ldots e^{Ky}\ldots]=f(x,y)$$

then calcurate the premitive function of $f(x,y)$,

$$\partial_x\partial_y F(x,y)=f(x,y)$$

$$F(x,y)=\mathrm{tr}[\ldots\frac{e^{Kx}}{K}\cdots\frac{e^{Ky}}{K}\ldots]+C_1(x)+C_2(y)$$

Set $F(x,\infty)=F(\infty,y)=0$, and take a limit $x,y\to 0$. it is the answer:

$$F(0,0)=\mathrm{tr}[\ldots\frac{1}{K}\cdots\frac{1}{K}\ldots]$$

(We suppose that

the condition $F(x,\infty) = F(\infty, y) = 0$ is equal to setting $C_1(x) = C_2(y) = 0$, since we expect that $\mathrm{tr}[\ldots \frac{e^{K\infty}}{K} \ldots] = 0$.)

Result

- Correct double-brane tention.
- EOM contracted with itself is OK

## 2 Reduse $\#1/K$ by algebraic manuplations

Do algebraic simplifications first,

$$\begin{aligned}\mathrm{tr}[\Psi Q\Psi] =&\mathrm{tr}[Bc\frac{1}{K}cKc\frac{K}{K-1}c\frac{K^2}{K-1}] - \mathrm{tr}[Bc\frac{1}{K}cKc\frac{K^2}{K-1}c\frac{K}{K-1}] \\ &+ \mathrm{tr}[Bc\frac{1}{K}c\frac{K^2}{K-1}cKc\frac{K}{K-1}]\end{aligned} \tag{7}$$

$\hookrightarrow$ Only one $1/K$ in each team in RHS.

then integrate (using Schwinger parametrization)

$$\frac{1}{K} \to -\int_0^\Lambda e^{Kx}dx$$

Results

- Correct double brane tention.
- EOM contracted with itself is OK.

## Conclusion

For the solution below,

$$\Psi = \frac{1}{K} c \frac{K^2 B}{K-1} c,$$

- Exact energy calculation is possiple.
- $1/K$ Schwinger parametrization is not good.
- Some prescriptions reproduces the desired results.
  - use the primitive function
  - do the algebraic simplification first.