Resolving singularities in higher spin gravity

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work in progress with

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Overview

- Introduction

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- 3D AdS higher spin gravity

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- Resolving conical defects

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- Outlook

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- $SL(N,\mathbb{R}) \times SL(N,\mathbb{R})$ CS theory describes spins $2, 3, \ldots, N$
- asymptotic symmetry algebra is extended from Virasoro to $\mathcal{W}_N$ algebra (Campoleoni, Fredenhagen, Pfenniger, Theisen 2010)

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- Concrete AdS/CFT proposal (Henneaux, Rey; Gaberdiel, Gopakumar 2010): $\mathcal{W}_N$ coset minimal models

$$\frac{su(N)_k \times su(N)_1}{su(N)_{k+1}}$$

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- Many CFT states with $0 \leq L_0 \leq c/24$. Smooth conical defect solutions?

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- Higher spin black holes: horizon only in a certain gauge, wormhole-like in other gauges (Kraus, Gutperle; Ammon, Gutperle, Kraus, Perlmutter 2011)
- Can higher spins resolve singularities in a similar way?

3D higher spin gravity

- $SL(N,\mathbb{R}) \times SL(N,\mathbb{R})$ Chern-Simons formulation

$$S = S_{CS}[A] - S_{CS}[\widetilde{A}] \ ,$$

$$S_{CS}[A] = \frac{k}{4\pi}\,\mathrm{tr}\int_{\mathcal{M}}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right) \qquad k = \frac{l}{4G}$$

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- gauge invariance

$$\delta A = dA + [A, \lambda], \qquad \delta \widetilde{A} = d\widetilde{A} + [\widetilde{A}, \widetilde{\lambda}]$$

- choose the principal embedding of $sl(2,\mathbb{R}) \subset sl(N,\mathbb{R})$

$$L_0 = \frac{1}{2}\begin{pmatrix} (N-1) & 0 & \cdots & & 0 \\ \vdots & & (N+1-2i) & & \\ & & & \ddots & \\ 0 & \cdots & & & -(N-1) \end{pmatrix},$$

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- remaining generators $W_m^{(s)}, m = -(s-1), \ldots, s-1$. Transform in **2s-1** representations, $s = 3, \ldots, N$:

$$[L_i, L_j] = (i-j)L_{i+j}, \qquad [L_i, W_m^{(s)}] = (i(s-1)-m)W_{i+m}^{(s)}$$

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- other embeddings of $sl(2,\mathbb{R})$ lead to different spin content (Ammon, Gutperle, Kraus, Perlmutter; Campoleoni, Fredenhagen, Pfenniger 2011)

- generalized vielbein and spin connection

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$$\begin{aligned} \delta e &= d\xi + [\omega, \xi] + [e, \Lambda], \\ \delta\omega &= d\Lambda + [\omega, \Lambda] + \frac{1}{l^2}[e, \xi] \\ \xi &= \frac{l}{2}\left(\lambda - \widetilde{\lambda}\right), \qquad \Lambda = \frac{1}{2}\left(\lambda + \widetilde{\lambda}\right). \end{aligned}$$

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- metric-like fields (invariant under frame rotations)

$$g_{\mu\nu} = \frac{1}{\mathrm{tr} L_0^2}\mathrm{tr}\left(e_\mu e_\nu\right) , \quad \phi_{\mu\nu\rho} \sim \mathrm{tr}\left(e_{(\mu} e_\nu e_{\rho)}\right), \ldots$$

generalized diffeomorphisms act nontrivially on the metric

Smooth gauge fields

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- imposing boundary conditions + gauge-fixing

$$\begin{aligned} A &= b^{-1} a_+ \, b dx^+ + b^{-1} db, \qquad b = e^{(\rho-\rho_0)L_0} \\ \widetilde{A} &= b \tilde{a}_- \, b^{-1} dx^- + b \, db^{-1} \end{aligned}$$

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- will take $a_+, \tilde{a}_-$ constant. Trivial holonomy around ϕ-circle imposes

$$e^{2\pi a_+} = 1\ (-1) \qquad e^{2\pi\tilde{a}_-} = 1\ (-1) \qquad N \text{ even}$$

Singular conical defects

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- nontrivial CS holonomy $\Rightarrow$ gauge fields are not smooth

General conical defects

- Ansatz for CS connections $\left(W_i^{(2)} \equiv L_i , \qquad c_s = \sqrt{-\frac{\mathrm{tr} L_0^2}{\mathrm{tr} W_{-1}^{(s)} W_1^{(s)}}} \right)$

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\begin{aligned}
a_+ &= \sum_{s=2}^{N} c_s \left(w_1^s W_1^{(s)} + w_{-1}^s W_{-1}^{(s)} \right) \\
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- conical defect metrics when

$$\Lambda = M^2 \quad \Leftrightarrow \quad \vec{w}_1 = \alpha \vec{w}_{-1}$$

gauge fields are of the form

$$a_+ = \begin{pmatrix} 0 & \alpha v_1 & & & & & \\ -v_1 & 0 & \alpha v_2 & & & & \\ & -v_2 & 0 & & & & \\ & & & \ddots & & & \\ & & & & 0 & \alpha v_{N-1} & (0) \\ & & & & -v_{N-1} & 0 & (0) \\ & & & & (0) & (0) & (0) \end{pmatrix}$$

Smooth conical defects

- Smooth $Sl(N,\mathbb{R})$ connection: $a_+, \tilde{a}_-$ have eigenvalues

$$\lambda_i = (in_1, -in_1, in_2, -in_2, \ldots, in_{[N/2]}, -in_{[N/2]}, (0)) \quad n_i \in \mathbb{Z} \text{ or } \mathbb{Z}/2$$

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- Can be realized as conical defects: choose $v_{2i-1} = n_i, v_{2i} = 0 \Rightarrow a_+, \tilde{a}_-$ have block form

$$a_+ = -\tilde{a}_- = \begin{pmatrix} B_1 & 0 & & & \\ 0 & B_2 & & & \\ & & \ddots & & \\ & & & B_{[N/2]} & (0) \\ & & & (0) & (0) \end{pmatrix} ; B_i = \begin{pmatrix} 0 & \frac{n_i}{\alpha} \\ -n_i\alpha & 0 \end{pmatrix}$$

- mass spectrum

$$M = -\frac{6}{N(N^2-1)} \sum_{i=1}^{[N/2]} n_i^2$$

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- energy spectrum and deficit angles

$$L_0 = \tilde{L}_0 = \frac{c}{24}(4M+1) \qquad \delta = 2\pi(1 - \sqrt{-4M})$$

- conical defect spectrum up to $N = 5$

N	(n_i)	M	$\frac{24}{c} L_0$	$\frac{\delta}{2\pi}$
2	$(1/2)$	$-1/4$	0	0
3	(1)	$-1/4$	0	0
4	$(3/2, 1/2)$	-1/4	0	0
	$(3/2, 0)$	-9/40	1/10	$1 - \sqrt{9/10}$
	$(1, 1)$	-1/5	1/5	$1 - \sqrt{4/5}$
	$(1, 0)$	-1/10	3/5	$1 - \sqrt{2/5}$
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- large N: $\mathcal{O}(N^{3N/2})$ conical defects.

Wormhole gauges

- Consider conical defects in block gauge. Perform upper-diagonal gauge transformation

$$A'_+ = g A_+ g^{-1}, \qquad \tilde{A}'_- = g^{-1} \tilde{A}_- g$$

$$g = \begin{pmatrix} g_1 & 0 & & & \\ 0 & g_2 & & & \\ & & \ddots & & \\ & & & g_{[N/2]} & (0) \\ & & & (0) & (1) \end{pmatrix}, \qquad g_i = \begin{pmatrix} 1 & \gamma_i \\ 0 & 1 \end{pmatrix}$$

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- ϕ circle doesn't pinch off anywhere

$$\frac{g_{\phi\phi}}{l^2} = \frac{6}{N(N^2-1)} \left(\sum n_i^2 (e^\rho - e^{-\rho})^2 + 2\alpha^2 \sum \gamma_i^2 n_i^2 \right)$$

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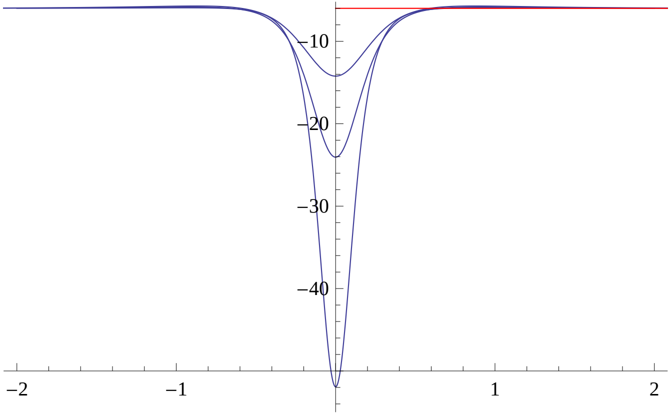
- scalar curvature
  $(N = 5, n_1 = n_2 = \alpha = 1, \gamma_1 = -\gamma_2 = 0, 0.2, 0.3, 0.4)$



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- Lead to reduced phase space (Drinfeld-Sokolov reduction) with classical $\mathcal{W}_N$ symmetry

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- Falloff conditions on gauge field $\rightarrow$ identify asymptotic symmetry algebra (a la Brown-Henneaux)
- Natural boundary conditions (Campoleoni, Fredenhagen, Pfenniger 2010)

$$(A - A_{AdS})_{|\rho\to\infty} = \mathcal{O}(1)$$

- Lead to reduced phase space (Drinfeld-Sokolov reduction) with classical $\mathcal{W}_N$ symmetry
- $\mathcal{W}_N$ survives on quantum level: captures vacuum representation (Gaberdiel, Gopakumar, Saha 2010)

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- Gauge-equivalent? u can be gauge-fixed such that

$$a_+ = \begin{pmatrix} 0 & u_1 & u_2 & \cdots & u_{N-1} \\ -1 & 0 & 0 & \cdots & 0 \\ & & \cdots & & \\ 0 & 0 & \cdots & -1 & 0 \end{pmatrix}$$

- If eigenvalues not degenerate, diagonalized by Vandermonde

$$V = \begin{pmatrix} \lambda_1^{N-1} & \lambda_2^{N-1} & \cdots & \lambda_N^{N-1} \\ & & \cdots & \\ \lambda_1 & \lambda_2 & \cdots & \lambda_N \\ 1 & 1 & \cdots & 1 \end{pmatrix}$$

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- Conical defects have degenerate eigenvalues $\rightarrow$ cannot be gauge-transformed into this form.
- Can falloff conditions be relaxed? E.g. include new sectors with falloff conditions

$$(A - A_{con.def.})_{|\rho\to\infty} = \mathcal{O}(1)$$

Other highest weight reps. of $\mathcal{W}_N$?

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  - encouraging: mass spectrum $\sim$ sum of squares of natural numbers (like Casimir of $su(N)$)
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- what is higher spin geometry?