Homotopy Operators and One-Loop Vacuum Energy at the Tachyon Vacuum

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§1. Introduction

(Bosonic) String field theory has classical solutions describing a tachyon vacuum where D-branes completely annihilate.

A numerical solution was constructed by using level truncation approximation in the Siegel gauge.

Sen-Zwiebach (99), Kostelecky-Samuel (90)

Analytic classical solutions were constructed.

Schnabl (05), Erler-Schnabl (09)

Identity-based solutions were considered earlier by many authors. One of them was constructed by Takahashi-Tanimoto (02), Kishimoto-Takahashi (02).

The solution has some properties of the tachyon vacuum.

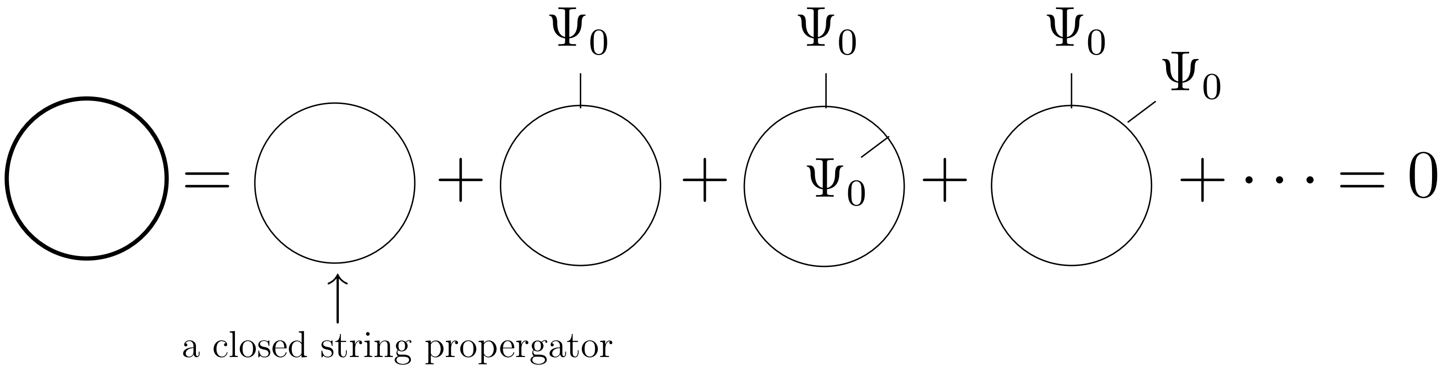
- Its vacuum energy cancels a D-brane tension.
- vanishing cohomology of the kinetic operator on its vacuum.

In SFT 2010 in Kyoto,
we reported numerical evaluation of vacuum loop amplitudes by the propagator in the theory expanded around the identity-based solution.

We find that the level truncated loop amplitude becomes zero as the identity-based solution approaches the tachyon vacuum solution ($a = -1/2$).

This result seems to be consistent with the fact that open strings disappear at the tachyon vacuum.

In other words, the numerical result implies that the brane decay process does not have any one-loop corrections.



This talk will present an analytic study of an one loop vacuum amplitude in the SFT expanded around the identity-based solution.

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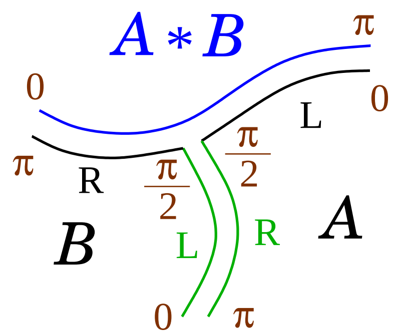
§2. Identity-based solutions in bosonic open SFT

String field theory

We consider bosonic open string field theory with a midpoint type interaction (Goto-Witten type).

Action

$$S[\Psi] = -\frac{1}{g^2}\int\left(\frac{1}{2}\Psi * Q_B\Psi + \frac{1}{3}\Psi * \Psi * \Psi\right)$$



Q_B: Kato-Ogawa BRST charge

Gauge symmetry

The action is invariant under the gauge transformation

$$\Psi' = e^{-\Lambda} * Q_B e^{\Lambda} + e^{-\Lambda} * \Psi * e^{\Lambda}$$

$$e^{\Lambda} = I + \Lambda + \frac{1}{2!}\Lambda * \Lambda + \frac{1}{3!}\Lambda * \Lambda * \Lambda + \cdots$$

I : identity string field

$I * A = A * I = A$ for $\forall\ A$

Equations of motion

$$Q_B \Psi + \Psi * \Psi = 0$$

Identity-based solutions

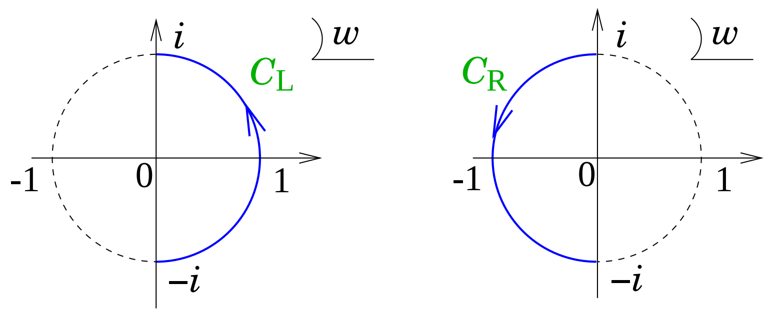
Takahashi-Tanimoto (02), Kishimoto-Takahashi (02), Takahashi-Zeze (03)

$$\Psi_0 = Q_L(e^h - 1)\, I - C_L((\partial h)^2 e^h)\, I$$

where we have defined the operators Q_L and C_L as

$$Q_L(f) = \int_{C_{\text{left}}} \frac{dw}{2\pi i} f(w)\, J_{\text{B}}(w), \quad C_L(g) = \int_{C_{\text{left}}} \frac{dw}{2\pi i} g(w)\, c(w).$$

The function $h(w)$ has to satisfy $h(-1/w) = h(w)$ and $h(\pm i) = 0$ in order that Ψ_0 may be a solution.



As a simple example, we consider an identity-based solution derived from the function,

$$h(z) = \log\left(1 + \frac{a}{2}\left(z + \frac{1}{z}\right)^2\right)$$

$$= -\log(1 - Z(a))^2 - \sum_{n=1}^{\infty} \frac{(-1)^n}{n} Z(a)(z^{2n} + z^{-2n}),$$

where $Z(a) = (1 + a - \sqrt{1 + 2a})/a$.

For the solution to be well defined, the parameter a is larger than or equal to $-1/2$.

This function generates the simplest expression of the theory expanded around the identity-based solution.

The solution is parametrized by a; $\quad \Psi_0(a) \quad (a \geq -1/2)$.

The solution has the following properties:

i) It has a well-defined universal Fock space expression.

$$|\Psi_0(a)\rangle = \varphi_0(a) c_1 |0\rangle + v_0(a) c_1 L^X_{-2} |0\rangle + u_0(a) c_{-1} |0\rangle + \cdots$$

ii) It can be represented as a trivial pure gauge configuration for $a > -1/2$, but it can't at $a = -1/2$.

$$\Psi_0(a) = g(a) * Q_B g^{-1}(a) \qquad \text{but } g(a) \text{ is singular at } a = -1/2!$$

The solution at $a = -1/2$ may be given as a kind of singular gauge transformation of the trivial configuration.

iii) No open strings on the vacuum of $a = -1/2$

Kishimoto-Takahashi (02)

If we expand the string field around the identity-based solution as $\Psi = \Psi_0(a) + \Phi$, the action for the fluctuation field Φ is given as

$$S[\Phi] = -\frac{1}{g^2}\int\left(\frac{1}{2}\Phi * Q'(a)\Phi + \frac{1}{3}\Phi * \Phi * \Phi\right),$$

where the kinetic operator is given by

$$Q'(a) = (1+a)Q_B + \frac{a}{2}(Q_2 + Q_{-2}) + 4aZ(a)c_0 - 2aZ(a)^2(c_2 + c_{-2})$$
$$-2a(1 - Z(a)^2)\sum_{n=2}^{\infty}(-1)^n Z(a)^{n-1}(c_{2n} + c_{-2n}).$$

To find spectrum on this vacuum, we have to consider the cohomology of the new BRST charge.

We can prove the following facts: Kishimoto-Takahashi (02)

1. For $a > -1/2$, the new BRST charge gives rise to the cohomology which has one-to-one correspondence to the cohomology of the original BRST charge. (The spectrum is unchanged.)

2. At $a = -1/2$, the new BRST charge has vanishing cohomology in the Hilbert space with the ghost number one.

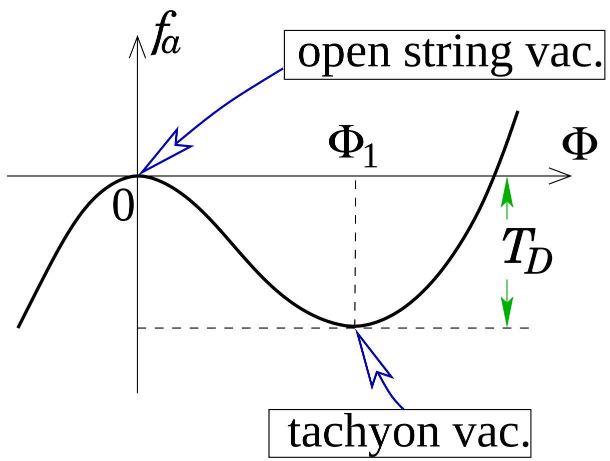
Therefore, we have no open string excitations in the theory expanded around $a = -1/2$ solution.

iv) Numerical analysis up to level 26 strongly suggests that the expanded theory has the vacuum structure as follows;

Kishimoto-Takahashi (09), Kishimoto (10)

For $a > -1/2$

(around a pure gauge configuration)

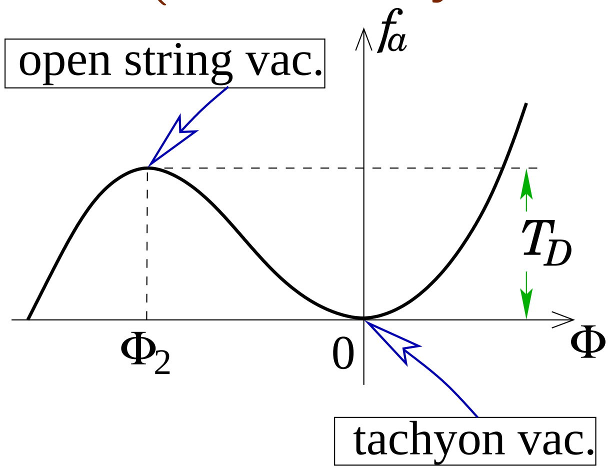


stable solution $\boldsymbol{\Phi}_1$

(tachyon vacuum)

At $a = -1/2$

(on the tachyon vacuum)



unstable solution Φ_2

(perturbative open string vacuum)

Vacuum energy of the unstable solution for the expanded theory at $a = -1/2$. Kishimoto-Takahashi (09), Kishimoto (10)

Level	Vacuum energy
(0,0)	2.3105795
(2,6)	2.5641847
(4,12)	1.6550774
(6,18)	1.6727496
(8,24)	1.4193393
(10,30)	1.4168893
(12,24)	1.3035715
(14,42)	1.2986472
(16,48)	1.2357748
(18,54)	1.2310583
(20,60)	1.1915648
(22,66)	1.1874828
(24,72)	1.1605884
(26,78)	1.1571287
$\cdots$	$\cdots$
$L \to \infty$	0.9909238

From all these results i)-iv), it is reasonable to expect that the identity-based solution $\Psi_0(a)$ can be regarded as

$a > -1/2 \;\cdots$ trivial pure gauge solution

$a = -1/2 \;\cdots$ tachyon vacuum solution

§3. Homotopy operators and identity-based solutions

We consider the theory expanded around the identity-based solution at $a = -1/2$:

$$S[\Phi] = -\frac{1}{g^2}\int\left(\frac{1}{2}\Phi * Q'\Phi + \frac{1}{3}\Phi * \Phi * \Phi\right),$$

where the kinetic operator is given by

$$Q' = Q(F) + C(G) = \frac{1}{2}Q_B - \frac{1}{4}(Q_2 + Q_{-2}) + 2c_0 + c_2 + c_{-2},$$

$$Q(F) = \oint \frac{dz}{2\pi i} F(z)\, j_B(z), \quad C(G) = \oint \frac{dz}{2\pi i} G(z)\, c(z),$$

$$F(z) = -\frac{1}{4}(z - 1/z)^2, \quad G(z) = z^{-2}(z + 1/z)^2.$$

To find a homotopy operators, we first calculate an anti-commutation relation;

$$\{Q(F),\, b(z)\} = \frac{3}{2}\partial^2 F(z) + \partial F(z) j_{gh}(z) + F(z)\, T(z)$$

$j_{gh}(z)$: ghost number current
$T(z)$: energy-momentum tensor

Since $F(z) = -(z - 1/z)^2/4$, by setting $z = \pm 1$, we find that

$$\{Q(F),\, b(\pm 1)\} = 1.$$

Note that the right hand side never becomes a c-number term if $a \neq -1/2$.

Therefore, the homotopy operator for Q' is given by

$$\hat{A} = \frac{1}{2}\{b(1) + b(-1)\}.$$

$\hat{A}$ is BPZ even and Hermitian, and satisfies

$$\{Q', \hat{A}\} = 1.$$

In general, we can find homotopy operators in the expanded theory around the identity-based solution which is expected to be the tachyon vacuum solution.

Note that we can not construct such homotopy operators for the identity-based solution corresponding to trivial pure gauge.

§4. One-loop vacuum amplitudes

First, we consider bosonic open string field theory on two parallel D-branes.

In the theory, the parameters of interbrane distance are included through zero-modes of string coordinates normal to the D-branes:

$$p^m = \frac{R'}{\pi}(\theta_j - \theta_i)$$

(X^m:normal directions, i, j: Chan-Paton indices)

Therefore, the tachyon vacuum solution depends on θ_i.

Then, the new BRST operator includes the interbrane distance parameters in the expanded theory around the tachyon vacuum solution.

Since Q' includes θ_i, a one-loop vacuum energy in this background seems to depend on the interbrane distances:

$$V \sim \int_0^\infty \frac{dt}{t} \, \mathrm{Tr} \left[(-1)^{N_{\mathrm{FP}}} e^{-tL'} b_0 c_0 \right], \qquad L' = \{Q', b_0\}.$$

fixing in the Siegel gauge

But, the vacuum energy is expected not to depend on the brane distance parameters, because the D-branes no longer exist at the tachyon vacuum!

Here, we will show that the vacuum energy is indeed independent of such moduli.

Under an infinitesimal change of moduli (for example $\delta\theta_i$),

$$Q' \to Q' + \delta Q'.$$

The variation of L' is given by

$$\delta L' = \{\delta Q', b_0\}.$$

The key ingredient of the proof is the existence of the homotopy operator $\widehat{A} = (b(1) + b(-1))/2$.

This $\widehat{A}$ satisfies

$$\{\widehat{A}, b_0\} = 0, \quad \{\delta Q', \widehat{A}\} = 0, \quad [L', \widehat{A}] = 0.$$

Now we are ready to evaluate the change of the partition function:

$$\delta Z(t) = -t \int_0^1 d\alpha \, \mathrm{Tr} \Big[(-1)^{N_{\mathrm{FP}}} e^{-\alpha t L'} \{\delta Q', \, b_0\} e^{-(1-\alpha) t L'} b_0 c_0 \Big]$$

Using $[L', \, b_0] = 0$ and the cyclic invariance of the trace, we find that

$$\mathrm{Tr} \Big[(-1)^{N_{\mathrm{FP}}} e^{-\alpha t L'} b_0 \, \delta Q' \, e^{-(1-\alpha) t L'} b_0 c_0 \Big]$$

$$= -\mathrm{Tr} \Big[(-1)^{N_{\mathrm{FP}}} e^{-\alpha t L'} \, \delta Q' \, e^{-(1-\alpha) t L'} b_0 c_0 b_0 \Big]$$

$$= -\mathrm{Tr} \Big[(-1)^{N_{\mathrm{FP}}} e^{-\alpha t L'} \, \delta Q' \, e^{-(1-\alpha) t L'} b_0 \Big] \, .$$

We insert $\{Q, \widehat{A}\}(=1)$ between $e^{-(1-\alpha)tL'}$ and b_0:

$$= -\mathrm{Tr}\left[(-1)^{N_{\mathrm{FP}}} e^{-\alpha t L'}\, \delta Q'\, e^{-(1-\alpha)tL'} \{Q', \widehat{A}\} b_0\right]$$

$$= -\mathrm{Tr}\left[(-1)^{N_{\mathrm{FP}}} e^{-\alpha t L'}\, \delta Q'\, e^{-(1-\alpha)tL'} Q'\, \widehat{A}\, b_0\right]$$

$$-\mathrm{Tr}\left[(-1)^{N_{\mathrm{FP}}} e^{-\alpha t L'}\, \delta Q'\, e^{-(1-\alpha)tL'} \widehat{A}\, Q'\, b_0\right].$$

In the second term, we move $\widehat{A}$ to the left by $\{\delta Q', \widehat{A}\} = 0$ and $[L', \widehat{A}] = 0$:

$$= -\mathrm{Tr}\left[(-1)^{N_{\mathrm{FP}}} e^{-\alpha t L'}\, \delta Q'\, e^{-(1-\alpha)tL'} Q'\, \widehat{A}\, b_0\right]$$

$$-\mathrm{Tr}\left[(-1)^{N_{\mathrm{FP}}} e^{-\alpha t L'}\, \delta Q'\, e^{-(1-\alpha)tL'}\, Q'\, b_0 \widehat{A}\right],$$

These two terms cancel each other thanks to $\{\widehat{A}, b_0\} = 0$.
Thus, we finally obtain

$$\delta Z(t) = 0.$$

§5. Homotopy operators and cohomology

Any state $|\psi\rangle$ satisfying $Q'|\psi\rangle = 0$ can be written as

$$|\psi\rangle = Q'|\phi\rangle \qquad (\ |\phi\rangle = \hat{A}|\psi\rangle \ )$$

On the other hand, the previous result (Kishimoto-T.T ('02)) is

$$|\psi\rangle = |\phi_0\rangle + Q'|\phi\rangle \qquad (\ |\phi\rangle = \hat{A}|\psi\rangle \ )$$

$$|\phi_0\rangle = |\mathrm{DDF}\rangle \otimes U b_{-2}|0\rangle + |\mathrm{DDF}\rangle \otimes U|0\rangle$$

$$U = \exp\left(-\sum_{n=1}^{\infty} \frac{2}{n} q_{-2n} \right)$$

The non-trivial state in the previous result also should be written as a BRST exact state:

$$|\phi_0\rangle = Q' \hat{A} \, |\phi_0\rangle \, .$$

The important fact is that $\hat{A} \, |\phi_0\rangle$ is a state out of the Fock space.

Actually, if we write it in the Fock space, the state becomes zero due to

$$\hat{A} \, U b_{-2} \, |0\rangle = \frac{1}{2} \{ b(1) + b(-1) \} U b_{-2} \, |0\rangle$$

$$= \exp \left(-2 \sum_{n=1}^{\infty} \frac{1}{n} \right) \frac{1}{2} U \{ b(1) + b(-1) \} b_{-2} \, |0\rangle = 0$$

Such a state is beyond the scope of the previous result.

This is not first appearance of such a state.

example 1. Bogoliubov transformations

The different vacua related by Bogoliubov transformations are orthogonal.

$$|0\rangle \;\leftarrow\; \text{Bogoliubov transformations} \rightarrow\; |0'\rangle$$

Then,

$$\langle 0\,|0'\,\rangle = 0\,!$$

Therefore, $|0'\rangle$ lives outside the Hilbert space on the original vacuum $|0\rangle$.

example 2. Dilaton condensation in SFT

In bosonic closed LCSFT, Yoneya constructed a classical solution representing dilaton vacuum expectation value (1987).

$$|\psi_0\rangle = -\frac{4a}{\sqrt{D-2}}\int_0^1 dt \exp\left(-\frac{2a}{D-2}\mathcal{D}t\right)|\phi_d\rangle\,,$$

$$\mathcal{D} = -\frac{i}{2}\{\hat{x},\,\hat{p}\} + \sum_{n\neq 0}\frac{1}{n}\alpha_n\tilde{\alpha}_n,$$

$$|\phi_d\rangle = \frac{1}{\sqrt{D-2}}\alpha_{-1}\cdot\tilde{\alpha}_{-1}\,|0\rangle\,\delta^{24}(p)\delta\alpha \quad (D=26).$$

Yoneya said “The transformation ($\psi = \psi_0 + \phi$) cannot be performed within a single Fock space...”.

In fact, $|\psi_0\rangle$ becomes zero if we write it in the Fock space.

$$|\psi_0\rangle = -\frac{4a}{\sqrt{D-2}} \int_0^1 dt \exp\left(-\frac{2a}{D-2}\mathcal{D}t\right) |\phi_d\rangle$$

$$= -\frac{4a}{\sqrt{D-2}} \int_0^1 dt \exp\left[-2 \log \cosh\left(\frac{2at}{D-2}\right) \cdot \frac{D-2}{2} \sum_{n=1}^{\infty} 1\right] \times$$

$$\times (\text{ normal ordered state })$$

$$= -\frac{4a}{\sqrt{D-2}} \int_0^1 dt\, e^{-\infty} \times (\text{ normal ordered state }) = 0!$$

example 3. Other classical solutions in SFT

Other classical solutions were constructed in the $\alpha = p_+$ HIKKO theory (Kugo-Zwiebach 1992). These solutions cannot be represented in the single Fock space.

In the paper, they said "Since we are dealing with a system with infinite number of degrees of freedom (oscillators) it turns out that different vacua, as related formally by Bogoliubov transformations are actually orthogonal. Their inner product is always zero! $\cdots$ We may be forced to admit that nontrivial classical solutions must live outside the Hilbert space of the original background, $\cdots$"

example 4. phantom terms

Original Schnabl's solution has a phantom term.

$$|\Psi_0\rangle = \lim_{N\to\infty}\left[\psi_N - \sum_{n=0}^{N}\partial_n\psi_n\right]$$

The phantom term becomes zero in a Fock space. Precisely, its contraction with Fock space states is always zero.

§6. Summary and discussion

We have constructed a homotopy operator $\hat{A}$ for the BRST operator Q' in the theory expanded around the identity-based solution.

Using the homotopy operator $\hat{A}$, we have demonstrated that the one-loop vacuum energy in the tachyon vacuum background is independent of moduli such as interbrane distances.

We can also prove $\delta Z(t) = 0$ for a part of other solutions in terms of the KBc subalgebra (for example, $\Psi = \sqrt{1-\beta K}\beta^{-1}\, c\sqrt{1-\beta K}$).

This result is consistent with numerical analysis, which was reported in SFT2010.

We have also revisited the cohomology problem for the identity-based solutions.

As a result, we conclude that there is no cohomology at all ghost numbers. However, the non-trivial cohomology part in the previous result cannot be regraded as a BRST exact state within a single Fock space. We have to incorporate the state outside a single Hilbert space.

"$\cdots$, but this will demand that we learn how to define string field theory beyond the usual methods based on oscillator expansions."

Kugo-Zwiebach (1987)