Level truncation of identity based solutions

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The KBc subalgebra

The simplest subalgebra relevant for tachyon condensation is obtained by star multiplication of the K, B and c

$$K = \frac{1}{2}\hat{\mathcal{L}} U_1^\dagger U_1 |0\rangle \, , \;\; B = \frac{1}{2}\hat{\mathcal{B}} U_1^\dagger U_1 |0\rangle \, , \;\; c = \frac{2}{\pi} U_1^\dagger U_1 c(0)|0\rangle \, ,$$

where $U_1^\dagger U_1 = e^{\frac{1}{2}\hat{\mathcal{L}}}$, and $\hat{\mathcal{L}}$, $\hat{\mathcal{B}}$ are given by

$$\begin{aligned} \hat{\mathcal{L}} &\equiv \mathcal{L}_0 + \mathcal{L}_0^\dagger = \oint \frac{dz}{2\pi i}(1+z^2)(\arctan z + \text{arccot} z)\, T(z) \, , \\ \hat{\mathcal{B}} &\equiv \mathcal{B}_0 + \mathcal{B}_0^\dagger = \oint \frac{dz}{2\pi i}(1+z^2)(\arctan z + \text{arccot} z)\, b(z) \, . \end{aligned}$$

The basic elements K, B and c of this subalgebra satisfy

$$\begin{aligned} &\{B, c\} = 1 \, , \qquad [B, K] = 0 \, , \qquad B^2 = c^2 = 0 \, , \\ &Q_B K = 0 \, , \qquad Q_B B = K \, , \qquad Q_B c = cKc \, . \end{aligned}$$

The identity based solution

Using the elements of the KBc subalgebra, we can find an identity based solution of the equations of motion $Q_B\Psi_I + \Psi_I\Psi_I = 0$

$$\Psi_I = c(1-K)\,.$$

Observations :

- The solution provides ambiguous result for the value of the vacuum energy therefore it is not a regular solution.
- The solution can be related to the well known Erler-Schnabl's solution Ψ_{ES} by a gauge transformation

$$\begin{aligned}\Psi_{ES} &= \mathcal{U}Q_B\mathcal{U}^{-1} + \mathcal{U}\Psi_I\mathcal{U}^{-1} \\ &= c(1+K)Bc\frac{1}{1+K}\,.\end{aligned}$$

$$\mathcal{U} = 1 + cBK$$

Gauge equivalence to a one-parameter family of solutions

From the previous last observation, we can consider instead a one-parameter gauge transformations (S. Zeze)

$$\mathcal{U}_\lambda = 1 + \lambda cBK \,, \qquad \mathcal{U}_\lambda^{-1} = 1 - \lambda cBK \frac{1}{1+\lambda K} \,.$$

So that we obtain

$$\begin{aligned} \Psi_\lambda &= \mathcal{U}_\lambda Q_B \mathcal{U}_\lambda^{-1} + \mathcal{U}_\lambda \Psi_I \mathcal{U}_\lambda^{-1} \\ &= c(1+\lambda K)Bc \frac{1+(\lambda-1)K}{1+\lambda K} \,. \end{aligned}$$

Comments about the solution $\mathbf{\Psi}_\lambda$:

- The limit case $\lambda \to 0$ corresponds to the identity based solution, while the case $\lambda \to 1$ corresponds to the Erler-Schnabl's solution.

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- It has been evaluated the vacuum energy for this solution Ψ_λ using two ways: analytically (S. Zeze) and using $\mathcal{L}_0$ level truncation computations (E. Arroyo). In the limit case $\lambda \to 0$, both ways provide the same answer for the vacuum energy.

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- So it has been claimed that Ψ_λ can be thought as a gauge invariant regularization of the identity based solution.
- In this presentation we would like to bring an additional support to this claim by analyzing the solution Ψ_λ from the traditional L_0 level truncation point of view.

L_0 level expansion of the regularized solution

After some algebraic manipulations, the regularized solution can be written as

$$\Psi_\lambda = \frac{2}{\pi}\int_0^\infty dt\, e^{-\frac{t}{\lambda}}\mathbf{O}_t\Big[\big(\frac{t+1}{2}\big)\mathcal{F}(t)U^\dagger_{t+1}c\Big(\tan\big(\frac{\pi t}{2t+2}\big)\Big)\Big]|0\rangle$$
$$+\,Q_B\Big\{\frac{\lambda}{\pi}\int_0^\infty dt\, e^{-\frac{t}{\lambda}}\mathbf{O}_t\Big[\mathcal{F}(t)U^\dagger_{t+1}\mathcal{B}^\dagger_0 c\Big(\tan\big(\frac{\pi t}{2t+2}\big)\Big)\Big]|0\rangle\Big\},$$

where

$$\mathbf{O}_t = \frac{1}{\lambda} - \frac{(\lambda-1)}{\lambda}\partial_t, \quad \mathcal{F}(t) = \cos^2\big(\frac{\pi t}{2t+2}\big), \quad U_r = (2/r)^{\mathcal{L}_0}.$$

As an example, let us expand Ψ_λ up to level two states

$$\Psi_\lambda = t_{(\lambda)} c_1|0\rangle + u_{(\lambda)} c_0|0\rangle + v_{(\lambda)} c_{-1}|0\rangle + w_{(\lambda)} L_{-2} c_1|0\rangle + \dots + Q_B\text{-exact},$$

where the coefficients $t(\lambda)$, $u(\lambda)$, $v(\lambda)$ and $w(\lambda)$ are given by

$$t(\lambda) = \frac{2}{\pi}\int_0^\infty dt\, e^{-\frac{t}{\lambda}}\Big(\frac{1}{\lambda} - \frac{(\lambda-1)}{\lambda}\partial_t\Big)\Big[\frac{1}{4}(t+1)^2\cos^2\Big(\frac{\pi t}{2t+2}\Big)\Big],$$

$$u(\lambda) = \frac{2}{\pi}\int_0^\infty dt\, e^{-\frac{t}{\lambda}}\Big(\frac{1}{\lambda} - \frac{(\lambda-1)}{\lambda}\partial_t\Big)\Big[\frac{1}{4}(t+1)\sin\Big(\frac{\pi t}{t+1}\Big)\Big],$$

$$v(\lambda) = \frac{2}{\pi}\int_0^\infty dt\, e^{-\frac{t}{\lambda}}\Big(\frac{1}{\lambda} - \frac{(\lambda-1)}{\lambda}\partial_t\Big)\Big[\sin^2\Big(\frac{\pi t}{2t+2}\Big)\Big],$$

$$w(\lambda) = \frac{2}{\pi}\int_0^\infty dt\, e^{-\frac{t}{\lambda}}\Big(\frac{1}{\lambda} - \frac{(\lambda-1)}{\lambda}\partial_t\Big)\Big[\frac{1}{12}(3-t^2-2t)\cos^2\Big(\frac{\pi t}{2t+2}\Big)\Big].$$

These integrals are convergent provided that the parameter λ belongs to the interval $(0, +\infty)$.

Evaluation of the vacuum energy

- Once we have the level expansion of the string field Ψ_λ, we can compute the normalized value of the vacuum energy. By performing the replacement $\Psi_\lambda \to z^{L_0}\Psi_\lambda$, we define

$$E_\lambda(z) \equiv \frac{\pi^2}{3}\langle z^{L_0}\Psi_\lambda, Q_B z^{L_0}\Psi_\lambda\rangle.$$

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- This replacement is necessary since we are going to evaluate the normalized value of the vacuum energy using Padé approximants.
- For instance, to evaluate the $P^n_{2+n}(\lambda, z)$ Padé approximants, we need to know the expansion of the normalized value of the vacuum energy $E_\lambda(z)$ up to the order z^{2n-2}. After doing our calculations, we simply set $z = 1$.

Results

Table 1: The Padé approximation for the normalized value of the vacuum energy $\frac{\pi^2}{3}\langle z^{L_0}\Psi_\lambda\ ,Q_B z^{L_0}\Psi_\lambda\ \rangle$ evaluated at $z=1$. The results show the P^n_{2+n} Padé approximation for various values of the parameter λ.

	$P^n_{2+n}[\lambda=\frac{1}{10}]$	$P^n_{2+n}[\lambda=\frac{2}{10}]$	$P^n_{2+n}[\lambda=\frac{3}{10}]$	$P^n_{2+n}[\lambda=\frac{4}{10}]$	$P^n_{2+n}[\lambda=\frac{5}{10}]$
$n=0$	-0.62053698	-0.52824370	-0.46182884	-0.41220084	-0.37384441
$n=4$	-1.87862775	-1.43543509	-1.19912518	-1.05497540	-0.96248549
$n=8$	-1.38144216	-1.26867724	-1.12709179	-1.03838541	-0.99164341
$n=12$	-1.37921868	-1.23797300	-1.11310799	-1.03746194	-0.85782931
	$P^n_{2+n}[\lambda=1]$	$P^n_{2+n}[\lambda=\frac{12}{10}]$	$P^n_{2+n}[\lambda=\frac{13}{10}]$	$P^n_{2+n}[\lambda=\frac{14}{10}]$	$P^n_{2+n}[\lambda=\frac{15}{10}]$
$n=0$	-0.26608479	-0.24232453	-0.23260344	-0.22399489	-0.21631929
$n=4$	-0.67935543	-0.52264016	-0.46558572	-0.41942773	-0.38129318
$n=8$	-0.93565531	-0.90981927	-0.89842271	-0.88782546	-0.87788963
$n=12$	-0.94057422	-0.90945338	-0.89948567	-0.88818475	-0.87439427

The real solution

- A real solution $\hat{\Psi}_\lambda$ can be generated by performing a gauge transformation on the previous solution Ψ_λ

$$\hat{\Psi}_\lambda = \sqrt{\frac{1+(\lambda-1)K}{1+\lambda K}}(\Psi_\lambda + Q_B)\sqrt{\frac{1+\lambda K}{1+(\lambda-1)K}}.$$

- This real solution can be written as

$$\hat{\Psi}_\lambda = \hat{\Psi}^{(1)}_\lambda + Q_B\hat{\Psi}^{(2)}_\lambda\,,$$

where

$$\hat{\Psi}^{(1)}_\lambda = \sqrt{\frac{1+(\lambda-1)K}{1+\lambda K}}\,c\,\sqrt{\frac{1+(\lambda-1)K}{1+\lambda K}},$$

$$\hat{\Psi}^{(2)}_\lambda = \sqrt{\frac{1+(\lambda-1)K}{1+\lambda K}}\lambda Bc\sqrt{\frac{1+(\lambda-1)K}{1+\lambda K}}.$$

We can expand the real solution in the basis of L_0 eigenstates

$$\hat{\Psi}_\lambda = t(\lambda)c_1|0\rangle + v(\lambda)c_{-1}|0\rangle + w(\lambda)L_{-2}c_1|0\rangle + \cdots + Q_B\text{-exact},$$

where the coefficients of the expansion $t(\lambda)$, $v(\lambda)$ and $w(\lambda)$ are

$$t(\lambda) = \frac{2}{\pi}\int_0^\infty ds\,dt\,\mathcal{I}(s,t,\lambda)\widehat{\mathcal{O}}_s\widehat{\mathcal{O}}_t\Big[\mathcal{J}(s,t)\Big],$$

$$v(\lambda) = \frac{2}{\pi}\int_0^\infty ds\,dt\,\mathcal{I}(s,t,\lambda)\widehat{\mathcal{O}}_s\widehat{\mathcal{O}}_t\left[\frac{4\mathcal{J}(s,t)\tan^2\left(\frac{\pi}{2}\frac{s-t}{s+t+1}\right)}{(s+t+1)^2}\right],$$

$$w(\lambda) = \frac{2}{\pi}\int_0^\infty ds\,dt\,\mathcal{I}(s,t,\lambda)\widehat{\mathcal{O}}_s\widehat{\mathcal{O}}_t\left[\frac{\big(4-(s+t+1)^2\big)\mathcal{J}(s,t)}{3(s+t+1)^2}\right].$$

$$\mathcal{I}(s,t,\lambda) = \frac{e^{-\frac{(s+t)(2\lambda-1)}{2(\lambda-1)\lambda}} I_0\left(\frac{1}{2(\lambda-1)\lambda}s\right) I_0\left(\frac{1}{2(\lambda-1)\lambda}t\right)}{(\lambda-1)\lambda}, \quad \mathcal{J}(s,t) = \left(\frac{s+t+1}{2}\right)^2\cos^2\left(\frac{\pi}{2}\frac{s-t}{s+t+1}\right),$$

$$\widehat{\mathcal{O}}_t = \left[1-(\lambda-1)\partial_t\right].$$

$I_0(x)$ is the modified Bessel function of the first kind.

These integrals are convergent provided that the parameter λ belongs to the interval $(1,+\infty)$.

Evaluation of the vacuum energy

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$$E_\lambda(z) \equiv \frac{\pi^2}{3}\langle z^{L_0}\hat{\Psi}_\lambda, Q_B z^{L_0}\hat{\Psi}_\lambda\rangle.$$

- This replacement is necessary since we are going to evaluate the normalized value of the vacuum energy using Padé approximants.
- For instance, to evaluate the $P^n_{2+n}(\lambda, z)$ Padé approximants, we need to know the expansion of the normalized value of the vacuum energy $E_\lambda(z)$ up to the order z^{2n-2}. After doing our calculations, we simply set $z = 1$.

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Table 2: The Padé approximation for the normalized value of the vacuum energy $\frac{\pi^2}{3}\langle z^{L_0}\hat{\Psi}_\lambda\ , Q_B z^{L_0}\hat{\Psi}_\lambda\ \rangle$ evaluated at $z=1$. The results show the P^n_{2+n} Padé approximation for various values of the parameter λ.

	$P^n_{2+n}[\lambda=\frac{11}{10}]$	$P^n_{2+n}[\lambda=\frac{12}{10}]$	$P^n_{2+n}[\lambda=\frac{13}{10}]$	$P^n_{2+n}[\lambda=\frac{14}{10}]$	$P^n_{2+n}[\lambda=\frac{15}{10}]$
$n=0$	-0.80293844	-0.76048459	-0.72392265	-0.69219068	-0.66442720
$n=4$	-0.75437392	-0.72667434	-0.70128551	-0.67758602	-0.65544662
$n=8$	-0.98896263	-0.98497033	-0.98008354	-0.97427446	-0.96760968
$n=12$	-0.99001621	-0.98660724	-0.98251492	-0.97749570	-0.97151457
	$P^n_{2+n}[\lambda=\frac{16}{10}]$	$P^n_{2+n}[\lambda=\frac{17}{10}]$	$P^n_{2+n}[\lambda=\frac{18}{10}]$	$P^n_{2+n}[\lambda=\frac{19}{10}]$	$P^n_{2+n}[\lambda=2]$
$n=0$	-0.63994910	-0.61821502	-0.59879290	-0.58133505	-0.56555878
$n=4$	-0.63480210	-0.61557936	-0.59769248	-0.58104857	-0.56555334
$n=8$	-0.96019541	-0.95214843	-0.94358184	-0.93459932	-0.92529280
$n=12$	-0.96462560	-0.95692661	-0.94853355	-0.93956627	-0.93013994

Conclusions

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- The evaluation of the vacuum energy by means of the L_0 level truncated solutions have confirmed the expected answer for the tachyon vacuum in agreement with Sen's first conjecture.
- Our results have provided an additional support to the fact that to extract a definite value for the vacuum energy, the identity based solution can be defined as the limit of a gauge equivalent one-parameter family of solutions.

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- The evaluation of the vacuum energy by means of the L_0 level truncated solutions have confirmed the expected answer for the tachyon vacuum in agreement with Sen's first conjecture.
- Our results have provided an additional support to the fact that to extract a definite value for the vacuum energy, the identity based solution can be defined as the limit of a gauge equivalent one-parameter family of solutions.
- We would like to extend this analysis to the case of the modified cubic superstring field theory, as well as to Berkovits open superstring field theory.